ALGEBRA EXAMINATION PAPERS

BRADBURY

ALGEBRA

EXAMINATION PAPERS

FOR ADMISSION TO

HARVARD, YALE, AMHERST, DARTMOUTH, BROWN,

AND TO THE

MASS. INSTITUTE OF TECHNOLOGY,

FROM

JUNE, 1878, TO SEPT. 1889 INCLUSIVE.

EDITED BY

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BOSTON:

THOMPSON, BROWN, AND COMPANY,

23 HAWLEY STREET.

University Press:
John Wilson and Son, Cambridge.

PREFACE.

THAT any single work on Algebra will contain *all* that may be demanded for admission to *all* the colleges is hardly to be expected. Yet many secondary schools are preparing pupils for several colleges, and to secure good results teachers must know the demands of all. For several of the colleges a very thorough training is necessary, and success cannot be secured without careful preparation and a great deal of practice in written work. For Harvard the requirement in Algebra was very greatly increased about 1878, and since that time it has stood at this high standard.

Any applicant who undertakes these Harvard papers without having been thoroughly trained by written examinations — limited in time as at the actual examination — is almost sure to fail.

This book contains the admission papers in Algebra of several leading colleges, and thus affords an opportunity for a comparison of requirements in Algebra that has never been given before. Here are over a thousand examples that have been given as tests by these colleges. No better list for practice can be found.

It should perhaps be said that in 1887 and 1888 Dartmouth had no printed examinations for admission, — as she received most of her candidates on certificate, — and that the papers inserted for those years were given to the classes after a few weeks' work in the College. The papers for Yale for September, 1884, and September, 1888, the compiler has been unable to obtain.

W. F. B.

CAMBRIDGE, October, 1889.

CONTENTS.

EXAMINATION PAPERS IN ALGEBRA FOR ADMISSION TO HARVARD COLLEGE.

JUNE, 1878.

1. Two workmen, A and B, are employed on a certain job at different wages. When the job is finished, A receives \$ 27, and B, who has worked three days less, receives \$ 18.75. If B had worked for the whole time, and A three days less than the whole time, they would have been entitled to equal amounts. Find the number of days each has worked, and the pay each receives per diem.

2. Find the value of x from the proportion

$$\left(\frac{10\sqrt[3]{a^2}}{3\sqrt[4]{b^5}}\right)^2 : x = \sqrt{\frac{5a\sqrt[3]{a^2}}{4\sqrt[5]{a^2.b^9}}} : \frac{9b^{-3}}{\sqrt{5}}.$$

Express the answer in its simplest form, free from negative and fractional exponents.

3. Simplify the expression

$$\frac{\dfrac{x^2+y^2}{x^2-y^2} - \dfrac{x^2-y^2}{x^2+y^2}}{\dfrac{x-y}{x+y} + \dfrac{x+y}{x-y}}.$$

4. Write out the first five terms and the last five terms of $(x-y)^{13}$.

5. Find the value of x from the equations

$$ax + by = l,$$
$$cy + dz = m,$$
$$ex + fz = n.$$

6. Find the greatest common divisor and the least common multiple of $6x^2 + 7x - 5$ and $2x^3 - x^2 + 8x - 4$.

7. Solve the equation

$$\frac{x + 13a + 3b}{5a - 3b - x} - \frac{a - 2b}{x + 2b} = 1.$$

SEPTEMBER, 1878.

1. Three men, A, B, C, are tried on a piece of work. It is found that A and B together can do a certain amount in 12 hours; B and C can do the same amount in 8 hours and 24 minutes; and C and A can do the same amount in 9 hours and 20 minutes. Find the time which each man would require to do the same amount singly.

2. Simplify $\dfrac{\dfrac{b-a}{x-b} - \left(\dfrac{a-2b}{x+b} - \dfrac{3x(a-b)}{x^2-b^2}\right)}{\dfrac{b^3+3b^2x+3bx^2+x^3}{x^3-b^3} \div \dfrac{(x+b)^2}{x^2+bx+b^2}}$.

3. Write out the first five terms and the last five terms of $(x-y)^{31}$.

4. Solve the equations, $\frac{1}{2}(x-y) = x - 4$, $xy = 2x + y + 2$.

5. Solve the equation, $x + \dfrac{1}{x} = 1 + \dfrac{1}{1 - \dfrac{4b^2}{a^2+3b^2}}$.

6. Find the value of x from the proportion,

$$\frac{3a}{2b^2} : \frac{1}{12b\sqrt{a}}\left(\frac{a}{\sqrt[3]{b}}\right)^2 = x : \sqrt{\left(\frac{b}{\sqrt[6]{16a}}\right)^3}.$$

Find a result free from fractional and negative exponents, and in the most reduced form.

7. Find the greatest common divisor and the least common multiple of $4x^4 - x^2 - 6x - 9$ and $8x^4 - 2x^3 - 7x^2 - 6x - 9$.

JUNE, 1879.

1. Solve the equation,

$$\frac{2ax-4b}{bx-a}-\frac{bx-a}{2ax-b}=\frac{2abx}{2abx^2-(2a^2+b^2)x+ab}.$$

Reduce the answers to their simplest forms.

2. Solve the equations,

$$\frac{5y}{x}+\frac{2y+3}{x+y}=0,$$
$$4x+3y=1.$$

State clearly what values of x and y go together.

3. Find the value of x from the proportion

$$\frac{3b}{4}\sqrt[12]{\frac{a^8b^{18}}{c^3}}:x=\frac{2(3ac)^2}{\sqrt[15]{(b^9c^5)}}:\frac{2\sqrt{a}}{bc}.$$

4. Simplify the fraction

$$\frac{\dfrac{1}{x-y}-\dfrac{x}{x^2-y^2}-\dfrac{y}{x^2+y^2}}{\left(\dfrac{x+y}{x-y}\right)^2-\left(\dfrac{x-y}{x+y}\right)^2}.$$

5. Find the greatest common divisor of

$$2x^2-3x+1 \text{ and } 2x^2-x-1.$$

6. Put the following question into equations:—

A and B walk for a wager on a course of one mile (5280 feet) in length. At the first heat, A gives B a start of 45 seconds, and beats him by 110 feet. At the second heat, A gives B a start of 484 feet, and is beaten by 6 seconds. Required, the rates at which A and B walk.

SEPTEMBER, 1879.

1. Several friends, on an excursion, spent a certain sum of money. If there had been 5 more persons in the party, and each person had spent 25 cents more, the bill would have amounted to $33. If there had been 2 less in the party, and each person had spent 30 cents less, the bill would have amounted to only $11. Of how many did the party consist, and what did each spend? Find all possible answers.

2. Solve the equations,

$$\begin{aligned} 2x + 4y + 27z &= 28, \\ 7x - 3y - 15z &= 3, \\ 9x - 10y - 33z &= 4. \end{aligned}$$

3. Solve the equation,

$$\frac{x+3b}{8a^2-12ab} + \frac{3b}{4a^2-9b^2} = \frac{a+3b}{(2a+3b)(x-3b)}.$$

Reduce the answers to their simplest forms.

4. Calculate the sixth term of

$$\left(\frac{\sqrt[3]{a}}{\sqrt[4]{2}\sqrt[11]{b^3}} - \frac{\sqrt{2}}{3a^4\sqrt[10]{b}}\right)^{27}.$$

Reduce the answer to its simplest form, cancelling all common factors of numerator and denominator, performing the numerical multiplications, and giving a result which has only one radical sign and no negative or fractional exponents.

5. Simplify the fraction

$$\frac{\dfrac{2x+y^3}{2x-y^3} + \dfrac{4x^2+y^6}{4x^2-y^6}}{\dfrac{2x-y^3}{2x+y^3} - \dfrac{8x^3-y^9}{8x^3+y^9}}.$$

6. Find the greatest common measure and the least common multiple of

$$4x^5 + 14ax^4 - 18a^3x^2 \text{ and } 24ax^3 + 30a^3x + 126a^4.$$

JULY, 1880.

1. Reduce to its simplest form $\dfrac{1}{x + \dfrac{2}{1 - \dfrac{x - 2}{2x + 1}}}$.

2. Divide $6x^{m+3n} - 19x^{m+2n} + 20x^{m+n} - 7x^m - 4x^{m-n}$ by $3x^{2n} - 5x^n + 4$.

3. Find the fourth term of $\left(\dfrac{2\sqrt{a}}{3} - \dfrac{6\sqrt[3]{b^2}}{a}\right)^{21}$, reducing it to its simplest form.

4. Find the greatest common measure and the least common multiple of $2x^5 - 11x^2 - 9$ and $4x^5 + 11x^4 + 81$.

5. A man walks 2 hours at the rate of $4\frac{1}{2}$ miles per hour. He then adopts a different rate. At the end of a certain time, he finds that if he had kept on at the rate at which he set out, he would have gone three miles further from his starting-point; and that if he had walked three hours at his first rate and half an hour at his second rate, he would have reached the point he has actually attained. Find the whole time occupied by the walk and his final distance from the starting-point.

6. Solve the equation

$$\frac{a}{b(2x-1)} - \frac{b(2x+1)}{a(x^2-1)} = \frac{1}{(2x-1)(x+1)} + \frac{1}{(2x-1)(x-1)}.$$

Reduce the answers to their simplest forms.

SEPTEMBER, 1880.

1. Reduce to its simplest form as one fraction

$$\left(\frac{x+y}{x-y} + \frac{x^2+y^2}{x^2+y^2}\right) \div \left(\frac{x-y}{x+y} - \frac{x^3-y^3}{x^3+y^3}\right).$$

2. Find the greatest common measure and the least common multiple of

$$x^6+3x^5+3x^4+9x^3-4x^2-12x \text{ and } x^6+3x^5-x^3-3x^2.$$

3. Find the sixth term of $\left(\frac{a\sqrt{a}}{\sqrt[3]{b^2}}-6\sqrt{b^3}\right)^{17}$, reducing the literal part of the term to its simplest form, and the numerical part into its prime factors.

4. A reservoir, supplied by several pipes, can be filled in 15 hours, every pipe discharging the same fixed number of hogsheads per hour. If there were 5 more pipes, and every pipe discharged per hour 7 hogsheads less, the reservoir would be filled in 12 hours. If the number of pipes were 1 less, and every pipe discharged per hour 8 hogsheads more, the reservoir would be filled in 14 hours. Find the number of pipes and the capacity of the reservoir.

5. Solve the equation

$$\frac{2x+1}{b}-\frac{3x+1}{a}=\frac{1}{x}\left(\frac{1}{b}-\frac{2}{a}\right).$$

Reduce the answers to their simplest forms.

JUNE, 1881.

1. What are the factors of x^3+y^3? Reduce to its simplest form the product of $\frac{x^2-y^2}{x^3+y^3}$ and $\frac{\frac{x^2+y^2}{y}-x}{\frac{1}{y}-\frac{1}{x}}$.

2. Solve the equation $\frac{1}{1+a+x}=1+\frac{1}{a}+\frac{1}{x}$.

3. Find the square root of

$$1+\sqrt{16x^m}+10x^m+12\sqrt{x^{3m}}+9x^{2m}.$$

4. What is meant by the expression $a^{\frac{3}{4}}$?

5. Solve the equation $\sqrt{x-8} - \sqrt{x-3} = \sqrt{x}$.

6. A man rows down a stream, of which the current runs $3\frac{1}{2}$ miles an hour, for $1\frac{2}{3}$ hours. He then rows up stream for $6\frac{1}{2}$ hours, and finds himself two miles short of his original starting-place. Find his rate and the distance he rowed down stream.

7. Find the 4th and the 14th term of $(2a - b)^{16}$.

SEPTEMBER, 1881.

1. Solve the equation

$$\frac{4a^2}{x+2} + \frac{4a^2 - b^2}{x(x^2-4)} = \frac{b^2}{x-2}.$$

2. Multiply

$$\frac{x+y}{x-y} - \frac{x-y}{x+y} - \frac{4y^2}{x^2-y^2} \text{ by } \frac{x+y}{2y}.$$

3. Solve the equations

$$\frac{m}{x} + \frac{n}{y} = a; \quad \frac{n}{x} + \frac{m}{y} = b.$$

4. Find the greatest common measure of

$$x^4 - 115x + 24 \text{ and } 24x^4 - 115x^3 + 1.$$

5. A man bought a number of railway shares when they were at a certain rate per cent discount for \$ 8500; and afterwards, when they were at the same rate per cent premium, he sold all but 20 of them for \$ 9200. How many did he buy, and what did he give for each of them?

6. Find the last four terms of

$$(a^{\frac{1}{2}} - 2b^{\frac{1}{3}})^{20},$$

reducing the numerical part of each term to its prime factors.

JUNE, 1882.

1. Simplify

$$\frac{a^2-bc}{(a-b)(a-c)}+\frac{b^2+ca}{(b+c)(b-a)}+\frac{c^2+ab}{(c-a)(c+b)}.$$

2. A man bought a certain number of sheep for \$ 300; he kept 15 sheep, and sold the remainder for \$ 270, gaining half a dollar a head. How many sheep did he buy, and at what price?

3. Find the greatest common divisor of

$$2x^5-11x^2-9 \quad \text{and} \quad 4x^5+11x^4+81.$$

4. Solve the equation $\dfrac{(4a^2-b^2)(x^2+1)}{4a^2+b^2}=2x.$

Reduce the answers to their lowest terms.

5. Find the square root of

$$x^3+2x^{\frac{5}{2}}-3x^2-4x^{\frac{3}{2}}+4x.$$

6. A and B can do a piece of work in 18 days; A and C can do it in 45 days; B and C in 20 days. Find the time in which A, B, and C can do it, working together.

SEPTEMBER, 1882.

1. Simplify $\dfrac{x-3a+\dfrac{4a^2}{a+x}}{x-\dfrac{2a^2}{a+x}}.$

2. Solve the equations $\left\{\begin{aligned}\frac{x}{a}+\frac{y}{b}&=1\\ \frac{a}{x}+\frac{b}{y}&=4\end{aligned}\right\}$

3. Find the factors of the least common multiple of

$$3x^5+2x^4+x^2 \quad \text{and} \quad 3x^4+2x^3-3x^2+2x-1.$$

4. Solve the equation

$$(3 + b^2)(x^2 - x + 1) = (3b^2 + 1)(x^2 + x + 1).$$

5. Find the terms which do not contain radicals in the development of

$$\left(\sqrt{2a} - \sqrt{\frac{b}{a}}\right)^4.$$

6. A hires a certain number of acres for \$420. He lets all but four of them to B, receiving for each acre \$2.50 more than he pays for it. The whole amount received from B is \$420. Find the number of acres.

7. Which is the larger, $\sqrt[3]{10}$ or $\sqrt[5]{46}$? Give the reason for your answer.

JUNE, 1883.

1. Solve the equation

$$\frac{1}{x} = 2 - \frac{4ax^2 - 3b(x - 2)}{2a(x^2 + 1) + 3b}.$$

2. A man walks, at a regular rate of speed, on a road which passes over a certain bridge, distant 21 miles from the point which the man has reached at noon. If his rate of speed were half a mile per hour greater than it is, the time at which he crosses the bridge would be an hour earlier than it is. Find his actual rate of speed, and the time at which he crosses the bridge.

Explain the *negative* answer.

3. Find the prime factors of the coefficient of the 6th term of the 19th power of $(a - b)$. What are the exponents in the same term, and what is the sign?

4. Reduce the following fraction to its lowest terms:

$$\frac{x^4 + 2x^2 + 9}{x^4 - 4x^3 + 10x^2 - 12x + 9}.$$

5. Prove that, if $a : b = c : d$,

$$\frac{a+b}{c+d} = \frac{a-b}{c-d} = \frac{a}{c} = \frac{b}{d}.$$

6. Solve the equations

$$xy = 4 - y^2,$$
$$2x^2 - y^2 = 17.$$

Find all the answers, and show what values of x and y belong together.

SEPTEMBER, 1883.

1. A man setting out on a journey drives, at the rate of a miles per hour, to the nearest railway station, distant b miles from his house. On arriving at the station, he finds that the express-train for his place of destination has left c hours before. At what rate should he have driven in order to reach the station just in time for the express-train?

Having obtained the general solution of this question, find what the answer becomes in the following cases:

(1) when $c = 0$; (2) when $c = \frac{b}{a}$; (3) when $c = -\frac{b}{a}$.

In case (2), how much time does the man have to drive from his house to the station?

In case (3), what is the meaning of the *negative* value of c?

2. Solve the equation

$$(2x - 1)^{\frac{1}{2}} - (3x + 1)^{\frac{1}{2}} = (x - 4)^{\frac{1}{2}}.$$

3. Solve the equation

$$\frac{ax}{a^2x - 2} - \frac{1}{a}\left(\frac{x - 3}{a^2x - 2} - \frac{1}{x}\right) = \frac{2}{2x - a^2x^2};$$

reducing the answers to their lowest terms. What do the answers become, if $a = -1$?

4. Reduce the following fraction to its lowest terms:

$$\frac{6x^5 - 2x^4 - 11x^3 + 5x^2 - 10x}{9x^5 + 3x^4 - 11x^3 + 9x^2 - 10x}.$$

5. What is the value of a^0? of a^{-n}? Give the *reasons*.

6. Solve the equations

$$xy + 4 = 0,$$
$$9x^2 - y^2 = 7.$$

Find all the answers, and state what values of x and y belong together.

JUNE, 1884.

1. A landowner laid out a rectangular lot containing 1200 square yards. He afterwards added 3 yards to one dimension of his lot, and subtracted $1\frac{1}{2}$ yards from the other; thereby increasing the area of his lot by 60 square yards. Find the dimensions of the lot before and after the change.

How do you explain the *negative* solution?

2. Solve the equation

$$\frac{x+1}{c} - \frac{2}{cx} = \frac{x+2}{ax - bx};$$

reducing the answers to their simplest forms.

3. Solve the equations

$$x^2 + y^2 = 52; \quad xy + 24 = 0.$$

Find all the sets of answers, and state which answers belong together.

4. Multiply $a^{\frac{3}{2}} - a^{\frac{1}{2}} + 1 - a^{-\frac{1}{2}} + a^{-\frac{3}{2}}$ by $a^{\frac{1}{2}} + 1 + a^{-\frac{1}{2}}$.

Simplify the following expression:

$$\sqrt[26]{\left[\sqrt[3]{x^2} \cdot \sqrt{\left(\frac{\sqrt{x}}{\sqrt[3]{x}}\right)^5}\right]^3}.$$

5. Prove that if the corresponding terms of two proportions be multiplied together, the result is a proportion.

6. Find the greatest common divisor of

$9x^5 - 7x^3 + 8x^2 + 2x - 4$ and $6x^4 - 7x^3 - 10x^2 + 5x + 2$.

SEPTEMBER, 1884.

(Time allowed, 1½ hours.)

1. Solve the equations

$$2x^2 + 3xy - 3y^2 + 12 = 0,$$
$$3x + 5y + 1 = 0,$$

and state what values of x and y belong together.

2. Solve the equation $\dfrac{a-c}{x-a} - \dfrac{x-a}{a-c} = \dfrac{3b(x-c)}{(a-c)(x-a)}$, reducing the results to their simplest form.

3. Find the sixth term of the 19th power of $\left(\sqrt[3]{x^2} - \dfrac{y^3}{2x}\right)$, reducing the result to its simplest form.

4. Find the greatest common divisor of

$2x^4 - 3x^3 + 2x^2 - 2x - 3$ and $4x^4 + 3x^2 + 4x - 3$.

5. Solve the equation

$$\sqrt{7-x} + \sqrt{3x+10} + \sqrt{x+3} = 0.$$

6. A vessel is half full of a mixture of wine and water. If filled up with wine, the ratio of the quantity of wine to that of water is ten times what it would be if the vessel were filled up with water. Find the ratio of the original quantity of wine to that of water.

JUNE, 1885.

(Time allowed, 1½ hours.)

1. Three students, A, B, and C, agree to work out a series of difficult problems, in preparation for an examination; and each student determines to solve a fixed number every day. A solves 9 problems per day, and finishes the series 4 days before B; B solves 2 more problems per day than C, and

finishes the series 6 days before C. Find the number of problems, and the number of days given to them by each student.

2. Solve the following equation, reducing the answers to their simplest form:

$$\frac{2}{1+3x} - \left(\frac{a(1+2x)}{b(1+3x)} - \frac{b(3x-1)}{a(2x+1)} \right) = 0.$$

3. Solve the equation $\dfrac{\sqrt{3}}{\sqrt{2x-1} - \sqrt{x-2}} = \dfrac{1}{\sqrt{x-1}}$.

4. A certain whole number, composed of three digits, has the following properties: 10 times the middle digit exceeds the square of half the sum of the digits by 21; if 99 be added to the number, the order of the digits is inverted; and if the number be divided by 11, the quotient is a whole number of two digits, which are the same as the first and last digits of the original number. Find the number.

5. Given $\dfrac{x+6y}{7x-2y} = 8$; find the value of $\dfrac{10x-3y}{2x-y}$.

6. Find the greatest common divisor of

$3x^4 - x^3 - 2x^2 + 2x - 8$ and $6x^3 + 13x^2 + 3x + 20$.

7. Find the square root of

$4 - 12x + 5x^2 + 26x^3 - 29x^4 - 10x^5 + 25x^6$.

SEPTEMBER, 1885.

(Time allowed, 1½ hours.)

1. A certain manuscript is divided between A and B to be copied. At A's rate of work, he would copy the whole manuscript in 18 hours; B copies 9 pages per hour. A finishes his portion in as many hours as he copies pages per hour; B is occupied 2 hours more than A upon his portion. Find the number of pages in the manuscript, and the numbers of pages in the two portions.

2. Solve the following equation, reducing the answers to their simplest form:

$$\frac{\frac{1}{2}(x-a)}{b(x+a)} = \frac{1}{a} - 2\,\frac{b - \left(x - \frac{2b^2}{a}\right)}{(x+a)^2}.$$

3. Solve the equations

$$\frac{3\sqrt{x} + 2\sqrt{y}}{4\sqrt{x} - 2\sqrt{y}} = 6, \qquad \frac{x^2+1}{16} = \frac{y^2-64}{x^2};$$

finding all the values of x and y, and showing which values belong together.

4. Two casks, of which the capacities are in the ratio of a to b, are filled with mixtures of water and alcohol. If the ratio of water to alcohol is that of m to n in the first cask, and that of p to q in the second cask, what will be the ratio of water to alcohol in a mixture composed of the whole contents of the two casks? Reduce the answer to its simplest form.

What does the answer (in its simplest form) become, if $m = q = 0$? and what is the simplest statement of the question in this case?

5. Find the 10th term of $(x-y)^{27}$; of $\left(\frac{9a}{\sqrt{b}} - \frac{2b}{\sqrt{a}}\right)^{27}$.

The numerical coefficients are not to be computed, but expressed in terms of their prime factors; the literal parts are to be reduced to the simplest form.

NOTE. The above five questions constitute the paper; and all applicants are expected to do them if possible. The following question is not required, and is not necessary to make a perfect exercise; but it may be added, at the discretion of the student, and will be counted to improve the quality of an imperfect exercise.

6. Reduce to its lowest terms

$$\frac{6x^5 - 9x^4 + 11x^3 + 6x^2 - 10x}{4x^5 + 10x^4 + 10x^3 + 4x^2 + 60x}.$$

JUNE, 1886.

(Time allowed, 1½ hours.)

1. A boat's crew, rowing at half their usual speed, row three miles down a certain river and back again, in the middle of the stream, accomplishing the whole distance in 2 hours and 40 minutes. When rowing at full speed, they go over the same course in 1 hour and 4 minutes. Find (in miles per hour) the rate of the crew when rowing at full speed, and the rate of the current.

(Notice *both* solutions of this problem.)

2. Solve the equation

$$3\sqrt{x^3+17}+\sqrt{x^3+1}+2\sqrt{5x^3+41}=0.$$

Substitute the answers, when found, in the equation, and show in what manner the equation is satisfied.

3. Solve the equations

$$x+\frac{4y+1}{x+2y}=2(y+1), \qquad x+3y+1=0.$$

4. Solve the equation

$$\frac{(a+2b)x}{a-2b}=\frac{a^2}{a-2b}-\frac{4b^2}{x};$$

and reduce the answers to their simplest form.

5. Find the greatest common divisor and the least common multiple of $4x^3-4x^2-5x+3$ and $10x^2-19x+6$.

6. Find the 6th and the 25th terms of the 29th power of $(x-y)$; reducing the numerical coefficients to their prime factors, and not performing the multiplications.

Find the 6th term of the 29th power of $\left(\frac{\sqrt[3]{a}}{b}-\frac{b^2}{2a}\right)$; reducing exponents to their simplest form, and combining similar factors.

SEPTEMBER, 1886.

(Time allowed, 1½ hours.)

1. Solve the equation

$$\frac{\frac{1}{8}[2\,b\,(x+1)]^2}{4\,b\,x^3+5\,a\,x}-a\left(\frac{1}{x}-\frac{5\,a\,x-4\,b}{4\,b\,x^2+5\,a}\right)=0;$$

and reduce the answers to their simplest forms.

2. Solve the equation $x^{-3}-x^3=7\,(x^3+1)$.

3. A and B have 4800 circulars to stamp for the mail; and mean to do them in two days, 2400 each day. The first day, A, working alone, stamps 800 circulars, and then A and B together stamp the remaining 1600; the whole job occupying 3 hours. The second day, A works 3 hours, and B 1 hour; but they accomplish only $\frac{9}{10}$ of their task for that day. Find the number of circulars which each stamps per minute, and the length of time that B works on the first day.

4. Find the value of x from the proportion

$$\frac{5\,a\,c}{b^2}\sqrt[3]{a\,b^2}\;:\;\sqrt[4]{\frac{9\,c^3}{a^2}}=x\;:\;\frac{3\,a^2}{2}\sqrt{\frac{3\,c}{a\,b}};$$

and express the answer with the use of only one radical sign.

5. Given the three expressions

$$\begin{aligned}&2\,x^4+\;\;x^3-\;8\,x^2-\;\;x+\;6,\\&4\,x^4+12\,x^3-\;\;x^2-27\,x-18,\\&4\,x^4+\;4\,x^3-17\,x^2-\;9\,x+18;\end{aligned}$$

find the greatest common divisor and the least common multiple of the *first two* of these expressions; also those of *the whole group of three.*

JUNE, 1887.

(Time allowed, 1 hour.)

1. Solve the following equation:

$$\sqrt{x-3}+\sqrt{3\,x+4}+\sqrt{x+2}=0.$$

Find two answers, and verify the positive answer by showing that it satisfies the equation.

2. A broker sells certain railway shares for $3240. A few days later, the price having fallen $9 per share, he buys, for the same sum, 5 more shares than he had sold. Find the price and the number of shares transferred on each day.

3. Solve the following equation, finding four values of x:

$$x^4 + (2a^2 + 3ab - 2b^2)^2 = 5(a^2 + b^2)x^2.$$

4. Reduce the following expression to its simplest form as a single fraction:

$$\frac{\dfrac{1-x^3}{1+x^3} - \dfrac{1-x}{1+x}}{\dfrac{1+x^2}{1-x^2} + \dfrac{1+x}{1-x}}$$

SEPTEMBER, 1887.

(Time allowed, 1 hour.)

1. Solve the following equation, finding four values of x:

$$(x+a)(x-b) - \frac{a^2(x+a)}{x+b} - \frac{b^2(x-b)}{x-a} = \frac{3a^2b^2}{(x-a)(x+b)}.$$

2. At 6 o'clock on a certain morning, A and B set out on their bicycles from the same place, A going north and B south, to ride until 1½ P.M. A moved constantly northwards at the rate of 6 miles per hour. B also moved always at a fixed rate; but, after a while, he turned back to join A. Four hours after he turned, B passed the point at which A was when B turned; and, at 1½ P.M., when he stopped, he had reduced, by one half, the distance that was between them at the time of turning.

Find B's rate, the time at which he turned, the distance between A and B at that time, and the time at which B would have joined A if the ride had been continued at the same rates of speed. Find the answers for *both solutions*.

3. Find the sixth term of each of the following powers:

$$(x-y)^7; \qquad \left(\frac{6a^2}{7b\sqrt{b}} - \frac{b}{\sqrt{3a}}\right)^7.$$

4. Reduce the following fraction to its lowest term:

$$\frac{6x^4 - 13x^3 + 3x^2 + 2x}{6x^4 - 9x^3 + 15x^2 - 27x - 9}.$$

JUNE, 1888.

(Time allowed, 1 hour.)

1. Reduce the following expression to its simplest form as a single fraction:

$$\frac{\frac{1-x^2}{1+y}\left(\frac{x}{1+x} - 1\right)}{1 - \left(\frac{1}{1-y} - \frac{x^2+y^2-x+y}{1-y^2}\right)}.$$

2. Solve the following equations, finding, and reducing to their simplest forms, two sets of values of x and y:

$$(x + 3y) : (2x - y) = \left(\frac{1}{b} - \frac{7}{3a}\right) : \frac{2}{b},$$

$$x^2 = \tfrac{1}{2}(xy + 3ay + 18a^2).$$

What are the answers, when $a = 2$ and $b = -3$?

3. Two travellers, A and B, go from P to Q at uniform but unequal rates of speed. A sets out first, travelling on foot at the rate of 20 minutes for every mile. B follows, going 1 mile while A traverses the distance $\frac{PQ}{80}$. B overtakes and passes A, 8 miles from P; and when B reaches Q, he is 9 miles ahead of A. Find the distance PQ, and B's rate of speed in minutes to the mile.

(Obtain two solutions.)

4. Two men, working separately, can do a piece of work in x days and y days, respectively; find an expression for the time in which both can do it, working together.

A is 20 years old, and B is -2 years older; what is the age of B?

What are the values of x which satisfy the equation $x^2 = 3x$?

5. Write out $(x - y)^{11}$.

Find the square root of

$$4x^6 - 12x^5 + 5x^4 + 26x^3 - 29x^2 - 10x + 25.$$

SEPTEMBER, 1888.

(Time allowed, 1 hour)

1. Reduce the following expression to its lowest terms as a single fraction:

$$\frac{\dfrac{2x}{3}}{\dfrac{1}{x} - \dfrac{2x^3 + 11x^2 - 43x - 24}{14x^3 - 31x^2 - 31x - 6}}$$

2. Solve the following equations, finding, and reducing to their simplest forms, two sets of values of x and y:

$$\frac{a}{y + 4b} = \frac{2b}{x - y},$$

$$\frac{1}{(b - a)x} - \left(\frac{3}{(a + b)y} - \frac{1}{a^2 - b^2}\right) = 0.$$

What are the values of x and y, if $a = 3$ and $b = -1$?

3. Tristram is ten years younger than Launcelot; and the product of the ages they attained in 1870 is 96. Find the ages they attain in 1888.

(Two solutions.)

4. A sum of \$100 is put at compound interest at 4 per cent per annum for x years; find a formula for the amount.

5. Write out the first five terms and the last five terms of $(x - y)^{31}$.

Find and reduce to its simplest form the fifth term of

$$\left(a^3 b - \frac{3b^{-2}}{\sqrt{a^5}}\right)^{31}.$$

JUNE, 1889.

(Time allowed, 1 hour.)

1. Find the greatest common divisor and the least common multiple of

$$6x^4 - 5x^3 - 10x^2 + 3x - 10 \text{ and } 4x^3 - 4x^2 - 9x + 5.$$

2. Solve the following equations, finding and reducing to their simplest forms *two sets of values* of x and y:

$$\frac{1}{x+y} - \left(\frac{y}{a(x-y)} - \frac{x+6a}{x^2-y^2}\right) = 0,$$
$$y : (7x - 2y) = (b - a) : (2a - 9b).$$

What do the answers become, when $a = 6$ and $b = -2$?

3. A certain librarian spends every year a fixed sum for books. In 1886, the cost of his purchases averaged two dollars per volume; in 1887, he bought 300 more volumes than in 1886; and in 1888, 300 more volumes than in 1887. The average cost per volume was thirty cents lower in 1888 than in 1887. Find the number of volumes bought each year, and the fixed price paid for them. (Obtain *two solutions.*)

4. Find the *fourth term* of $(x - y)^{27}$;

of $$\left(\frac{b}{\sqrt[3]{a^2}} - \frac{5}{1}a^{\frac{5}{3}}b^{-2}\right)^{27}.$$

5. Solve the equation

$$\sqrt{(x+a)} + \sqrt{x} + \sqrt{(x-a)} = 0.$$

Explain the possibility of satisfying this equation, the connecting signs being both *plus*.

SEPTEMBER, 1889.

(Time allowed, 1 hour.)

1. Reduce the following expression to its simplest form as a single fraction: —

$$\frac{\left(a^2 + \frac{b^4}{a^2+b^2}\right)(a^2 - b^2)}{1 - \frac{b}{a+b} - \frac{1}{2}\left(1 - \frac{a-b}{a+b}\right)}.$$

2. Find the second term of $(x - y)^{51}$;

$$\text{of } \left(\sqrt[5]{a^3} - \frac{2}{17\,a^9}\right)^{51}.$$

Extract the square root of $4x^4 - 12x^3 - 11x^2 + 30x + 25$.

3. The distances traversed in any given time by two couriers, who are travelling on the same road in the same direction, are to each other in the ratio of p to q. The second courier passes a given point on the road n hours later than the first. How many hours after he passes this point will the second courier be with the first?

In what case is the answer negative, and what is the interpretation of this result?

4. A certain principal (x dollars), at simple interest at y per cent per annum for two years, earns a certain interest $\left(\frac{2xy}{100}\right)$. If the principal had been $(x - 20)$ dollars, and at compound interest at y per cent per annum, compounded annually, it would have earned the same interest in the same time. If the principal had been $(x + 80)$ dollars, and at simple interest at $(y - \frac{1}{2})$ per cent per annum, the interest for two years would have been \$1 less than that actually earned. Find the values of x and y.

5. Solve the following equations, finding and reducing to their simplest forms two sets of values of x and y: —

$$x + y : x - y = a : b,$$

$$\frac{a\,y}{b} - 4\,(a - b) = \frac{b}{a}\left(y - \frac{3\,a^2}{x}\right).$$

What do the answers become, when $a = -3$ and $b = 1$?

EXAMINATIONS FOR ADMISSION TO YALE COLLEGE.

JUNE, 1878.

1. (*a*) Reduce $\dfrac{a^3 + a^2x - ax^2 - x^3}{a^2 - x^2}$ to its lowest terms.

   (*b*) Multiply $a^{-6}b^2$ by $\dfrac{\sqrt{b}}{\sqrt[3]{a^2}}$; and divide $a^{-6}b^2$ by $\dfrac{\sqrt{b}}{\sqrt[3]{a^2}}$.

2. Solve the equations:

   (*a*) $\dfrac{7x-6}{35} - \dfrac{x-5}{6x-101} = \dfrac{x}{5}$.

   (*b*) $\dfrac{7x+9}{4} - \left(x - \dfrac{2x-1}{9}\right) = 7$.

3. (*a*) Solve the equation $\dfrac{x^2}{2} - \dfrac{x}{3} + 7\frac{3}{8} = 8$.

(*b*) It is required to find three numbers such that the product of the first and second may be 15, the product of the first and third 21, and the sum of the squares of the second and third 74.

4. Find the sum of n terms of the series 1, 2, 3, 4, 5, 6, &c.

5. By the binomial theorem expand to five terms $(a^3 - b^3)^{-\frac{1}{3}}$.

SEPTEMBER, 1878.

1. Find the value of each of the following expressions:

   (*a*) $\dfrac{1-x^2}{1+y} \times \dfrac{1-y^2}{x+x^2} \times \left(1 + \dfrac{x}{1-x}\right)$;

   (*b*) $(a^{-3} - a^{\frac{3}{2}})^2$;

   (*c*) $3\sqrt{\frac{2}{5}} + 2\sqrt{\frac{1}{10}} + 4\sqrt{\frac{1}{40}}$.

2. (*a*) $\frac{1}{x}+\frac{1}{y}-\frac{1}{z}=a\,;\ \frac{1}{x}-\frac{1}{y}+\frac{1}{z}=b\,;\ -\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=c.$
Find x, y, and z.

(*b*) Solve the equation

$$\frac{17-3x}{5}-\frac{4x+2}{3}=5-\left(6x-\frac{7x+14}{3}\right).$$

3. Solve the equations

(*a*) $$\frac{10}{x}-\frac{10}{x+1}=\frac{3}{x+2}.$$

(*b*) $$2x^{\frac{1}{3}}+3x^{\frac{1}{3}}=2.$$

4. (*a*) Find the sum of 13 terms of the series $2\frac{1}{2}$, $2\frac{5}{6}$, $3\frac{1}{6}$, etc.

(*b*) Find the value of $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}$, etc., to infinity.

5. By the binomial theorem expand to five terms $(a^2+x^2)^{-\frac{3}{2}}$.

JUNE, 1879.

1. Divide $(3a-b)$ by $a+b+\dfrac{a-b}{1+\dfrac{a-b}{a+b}}$ and simplify.

2. (*a*) Find the sum and difference of $\sqrt{18\,a^3b^3}$ and $\sqrt{50\,a^3b^3}$.

(*b*) Multiply $2\sqrt{3}-\sqrt{-5}$ by $4\sqrt{3}-2\sqrt{-5}$.

3. Solve the equation $\frac{x-1}{7}+\frac{23-x}{5}=7-\frac{4+x}{4}$.

4. Solve the equation $\frac{x-3}{x-2}-\frac{x-4}{x-1}=\frac{7}{20}$.

5. The sum of an arithmetical progression whose first term is 2 and last term 42, is 198; find the common difference and the number of terms.

6. Expand to four terms, by the binomial theorem, $(a^3-b^3)^{\frac{1}{3}}$.

SEPTEMBER, 1879.

1. Add together $\frac{1}{4a^6(a^2+x^2)}$, $\frac{1}{4a^6(a^2-x^2)}$, $\frac{1}{2a^4(a^4+x^4)}$.

2. (*a*) Multiply together $\frac{1}{2}\sqrt{3}$, $\frac{1}{3}\sqrt[3]{3}$, and $\frac{1}{4}\sqrt[6]{3}$.

   (*b*) Divide $9m^2(a-b)^{\frac{1}{2}}$ by $3m(a-b)^{\frac{1}{4}}$.

3. Solve the equations

   (*a*) $3x - \frac{x-4}{4} - 4 = \frac{5x+14}{3} - \frac{1}{12}$.

   (*b*) $\frac{1}{x}+\frac{1}{y}=a$; $\frac{1}{x}+\frac{1}{z}=b$; $\frac{1}{y}+\frac{1}{z}=c$.

4. Solve the equation

$$\frac{15}{x} - \frac{72-6x}{2x^2} = 2.$$

5. Find the sum of 20 terms of the series 1, 4, 10, 20, 35, &c.

6. By the binomial theorem expand to 4 terms

   (*a*) $(1-b)^{-\frac{3}{2}}$; (*b*) $(a^2-x^2)^{\frac{1}{2}}$.

JULY, 1880.

1. (*a*) Divide $\frac{a+1}{a-1}+\frac{a-1}{a+1}$ by $\frac{a+1}{a-1}-\frac{a-1}{a+1}$, and reduce the quotient to its simplest form.

   (*b*) Find the greatest common divisor of

$$x^4 - 6x^2 - 8x - 3 \text{ and } 4x^3 - 12x - 8.$$

2. (*a*) Find the sum of $6\sqrt[6]{4a^2}$, $2\sqrt[3]{2a}$, and $\sqrt[9]{8a^3}$.

   (*b*) Reduce to its simplest form the product

$$(x-1-\sqrt{-2})(x-1+\sqrt{-2})(x-2+\sqrt{-3})(x-2-\sqrt{-3}).$$

3. Solve the equations

(*a*) $\frac{1}{3}(2x - 10) - \frac{1}{11}(3x - 40) = 15 - \frac{1}{5}(57 - x)$;

(*b*) $x - 1 + \frac{2}{x - 4} = 0$;

(*c*) $\frac{x}{x^2 - 1} = \frac{x^2 + 1}{x}$.

4. Four numbers are in arithmetical progression; the product of the first and third is 27, and the product of the second and fourth is 72. What are the numbers?

5. By the binomial theorem expand to 4 terms,

(*a*) $(1 - b)^{-3}$; (*b*) $(x^2 - y^2)^{\frac{1}{3}}$.

SEPTEMBER, 1880.

1. (*a*) Required, in its simplest form, the quotient of

$$\frac{a^4 - x^4}{a^2 - 2ax + x^2} \div \frac{a^2x + x^3}{a^3 - x^3}.$$

(*b*) Find the greatest common divisor of

$$6x^2 - 17x + 12 \text{ and } 12x^2 - 4x - 21.$$

2. Find the sum of

$$\sqrt[4]{16},\ \sqrt[3]{81},\ -\sqrt[3]{-512},\ \sqrt[3]{192},\ -7\sqrt[6]{9}.$$

3. Solve the equations

(a) $5x - \frac{2x-1}{3} + 1 = 3x + \frac{x+2}{2} + 7$;

(b) $3x^2 + 10x - 57 = 0$;

(c) $\frac{x^2}{3} - \frac{x}{10} + \frac{1}{6} = \frac{1}{5}$.

4. Find three geometrical means between 2 and 162.

5. By the binomial theorem expand to 4 terms,

(a) $(1+a)^{\frac{1}{2}}$; (b) $\frac{1}{(a^8 - x^8)^{\frac{3}{8}}}$.

JULY, 1881.

1. Free from negative exponents $(4a^{-3}b^2x^{-4})^{-4}$.

2. Reduce to lowest terms $\frac{x^2 - 2x - 15}{x^2 + 10x + 21}$.

3. Factor $n^3 - 2n^2 + n$; $x^8 - 1$; $x^3 - n^3y^3$; $x^6 + y^6$.

4. Make denominator rational of $\frac{2}{\sqrt{5} - \sqrt{2}}$.

5. Multiply $\sqrt{x} - 2 + \sqrt{-3}$ by $\sqrt{x} + 2 - \sqrt{-3}$.

6. Solve $\frac{5}{x} - \frac{3x+1}{x^2} = \frac{1}{4}$.

7. Solve $x^2 - xy = 153$; $x + y = 1$.

8. By the Binomial Theorem expand to four terms $\frac{1}{\sqrt{n - x^2}}$

9. Sum the infinite series $1 + \frac{1}{2} + \frac{1}{4} +$ &c.

SEPTEMBER, 1881.

1. Find the greatest common divisor of

$$x^2 - 16 \text{ and } x^2 - x - 20.$$

2. Factor $x^4y + xy^4$, $x^8 - y^8$, $n^2x^3 + 8nx^2 + 16x$, $x^3 + 4x^2 + 4x + 2x + 4$.

3. Simplify $\dfrac{\dfrac{x}{xy} + \dfrac{y}{x+y}}{\dfrac{x}{x-y} - \dfrac{y}{x+y}}$.

4. Solve $x = \dfrac{2}{x + \sqrt{2 - x^2}} + \dfrac{2}{x - \sqrt{2 - x^2}}$.

5. Add $\sqrt[3]{16}$, $\sqrt[3]{54}$, and $\sqrt[3]{128}$.

6. Multiply $2 - \sqrt{-3} - 3\sqrt{-2}$ by $4\sqrt{-3} + 6\sqrt{-2}$.

7. Solve $\dfrac{x^3 - x^2 + 7}{x^2 + 3x - 1} = x + \dfrac{11}{3}$.

8. Expand $(x^2 - a)^{\frac{1}{2}}$ to four terms.

9. Given, in arithmetical progression, the first term, common difference, and sum of series; find last term.

JUNE-JULY, 1882.

1. Factor $a^3 - 4a^2b + 4ab^2$, $4x^4y^4 - 9x^2y^2$, $a^2 - 4b^2 + 4bc - c^2$.

2. Solve $x^2 = 21 + \sqrt{x^2 - 9}$.

3. Find the continued product of

$$x - (2 + \sqrt{3}),\ x - (2 - \sqrt{3}),\ x - (3 - \sqrt{-1}),\ x - (3 + \sqrt{-1}).$$

4. Divide 50 into two parts, such that the greater increased by 3, shall be to the less diminished by 3, as 3 to 2.

5. Given $\left\{\begin{matrix} x^2 + y^2 = 25 \\ 2xy = 24 \end{matrix}\right\}$, to find x and y.

6. Sum the infinite series 1, $\frac{1}{3}$, $\frac{1}{9}$, etc.

7. Resolve $\frac{5x - 19}{x^2 - 8x + 15}$ into partial fractions.

8. Expand by the binomial theorem to 3 terms $\frac{b}{a}\sqrt{x^2 - a^3}$.

9. Revert the series $y = x + x^2 + x^3 + x^4$.

SEPTEMBER, 1882.

1. Factor $x^8 - x^2y^6$, $x^2 + x + \frac{1}{4}$, $x^6z + 2x^3y^3z + y^6z$, $m^2 - a^2 + 2ab - b^2$.

2. Multiply $x - 2\sqrt{5} + 3\sqrt{-5}$ by $x - 2\sqrt{5} - 3\sqrt{-5}$.

3. Find x from the equation $\frac{1}{x-2} - \frac{1}{x-4} = \frac{1}{x-6} - \frac{1}{x-8}$.

4. Find x from $(\sqrt{x} - \sqrt{b})^{\frac{1}{2}} = nx^{\frac{1}{4}}$.

5. Find x from $\frac{x - 1}{x - 20} = 2x$.

6. The sum of the first and second of four numbers in geometrical progression is 15, and the sum of the third and fourth is 60. Required the numbers.

7. Expand $\frac{1}{3x - x^2}$ into an infinite series.

8. How many combinations can be made of 8 letters taken 5 at a time?

9. Extract the square root of $6 + \sqrt{20}$.

JUNE, 1883.

1. Reduce the following expression to its simplest form,

$$\frac{1}{x\,(x-a)\,(x-b)}+\frac{1}{a\,(a-x)\,(a-b)}+\frac{1}{b\,(b-x)\,(b-a)}.$$

2. Resolve $y^9 - b^9$ into three factors.

3. Change $x\,y^{-z} - 2\,x^{\frac{1}{2}}\,y^{-1}\,z^{-\frac{1}{2}} + z^{-1}$ to an expression which will contain no negative exponents.

4. If $\frac{a+b+c+d}{a+b-c-d} = \frac{a-b+c-d}{a-b-c+d}$, prove by the principles of proportion that $\frac{a}{b} = \frac{c}{d}$.

5. Find the value of $2\,a\,\sqrt{1+x^2}$ when

$$x = \frac{1}{2}\left(\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}}\right).$$

6. Given $(7 - 4\sqrt{3})\,x^2 + (2 - \sqrt{3})\,x = 2$, to find x.

7. The sum of two numbers is 16, and the sum of the reciprocals is $\frac{1}{3}$. What are the numbers?

8. Compute the value of the continued fraction

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{4+\frac{1}{5}}}}$$

9. Convert $\frac{1}{\sqrt{1+x^2}}$ into an infinite series by the method of Indeterminate Coefficients, or by the Binomial Theorem.

10. Insert three geometrical means between $\frac{1}{2}$ and 128.

SEPTEMBER, 1883.

1. Divide

$$\frac{c-b}{c+b}-\frac{c^2-b^2}{c^2+b^2} \text{ by } \frac{c+b}{c-b}+\frac{c^2+b^2}{c^2-b^2}.$$

2. Find the value of $\dfrac{1}{\sqrt{3}+1}$ to three decimal places.

3. Given $\sqrt{x^3}=2\sqrt{2}$, to find x.

4. Find the $-\frac{3}{4}$th power of $256\,x^3y^{-4}$.

5. A grocer has two sorts of tea, one worth a cents a pound, the other b cents a pound. How many pounds of each sort must be taken to make a mixture of m pounds worth c cents a pound?

6. Reduce $\dfrac{a^{m-\frac{1}{2}}}{b^{m+\frac{1}{2}}} \div \dfrac{a^{m+\frac{1}{2}}}{b^{m-\frac{1}{2}}}$ to its simplest form.

7. Solve the equation

$$4\sqrt[4]{x}+\sqrt{x}=21.$$

8. Find the cube root of

$$x^6-6x^5+3x^4+28x^3-9x^2-54x-27.$$

9. In the proportion

$$\frac{a^2-b^2}{b^2}=\frac{c^2-d^2}{d^2} \text{ prove that } \frac{a}{b}=\frac{c}{d}.$$

10. Insert three arithmetical means between -9 and 18.

11. Write the eleventh term of $(a-b)^{12}$.

JUNE, 1884.

1. Reduce the following fractions to their lowest terms:

$$\frac{a^2c+abc+b^2c}{a^4+a^2b^2+b^4}; \quad \frac{b^2+4y^2}{b^6+64y^6}.$$

2. Reduce $\frac{x+1}{x-1}\sqrt{\frac{x-1}{x+1}}$ to its simplest form.

3. Solve the equations

$$x+y=\frac{2(a^2+b^2)}{a^2-b^2}\,;\quad x-y=\frac{4ab}{a^2-b^2}.$$

4. Multiply $x-\sqrt{5}+1-\sqrt{-10-2\sqrt{5}}$ by

$$x-\sqrt{5}+1+\sqrt{-10-2\sqrt{5}}.$$

5. Find the number whose cube root is one fifth of its square root.

6. Find x from the equation $\sqrt{1+x-x^2}-2(1+x-x^2)=\frac{1}{3}$.

7. A and B can do a piece of work together in 8 days. A works alone 4 days, and then both finish it in 5 days more. In what time could each have done it alone?

8. A traveller has a journey of 132 miles to perform. He goes 27 miles the first day, 24 the second, and so on, travelling 3 miles less each day than the day before. In how many days will he complete the journey?

9. The ratio of the circumference of a circle to its diameter is 3.141592. Find by continued fractions three approximate values.

10. Expand by the method of undetermined coefficients to four terms $\sqrt{a-x^2}$.

JUNE, 1885.

1. Given $\frac{5x+2}{3} - \left(3 - \frac{3x-1}{2}\right) = \frac{3x+19}{2} - \left(\frac{x+1}{6} + 3\right)$, to find x.

2. Multiply $\frac{b-y}{a^3+y^3}$, $\frac{ca+cy}{b^2-by}$, $\frac{b^4+y^6}{b^2+y^2}$, and $\frac{b}{c}$.

3. Multiply $x - \frac{1}{2}(1-\sqrt{-3})$ by $x - \frac{1}{2}(1+\sqrt{-3})$.

4. Divide $x^2y^{-\frac{4}{3}} - 2 + x^{-2}y^{\frac{4}{3}}$ by $x^{\frac{1}{2}}y^{-\frac{1}{3}} - x^{-\frac{1}{2}}y^{\frac{1}{3}}$.

5. Given $91x^2 - 2x = 45$, to find both values of x.

6. Given $\frac{7}{\sqrt{x}} + \frac{4}{\sqrt{y}} = 4$,

$$\frac{1}{\sqrt{x}} + \frac{2}{\sqrt{y}} = 1, \text{ to find } x \text{ and } y.$$

7. Expand by the Binomial Theorem to five terms $(1+a)^{xy}$.

8. In Arithmetical Progression, given $d =$ the common difference, $a =$ the first term, and $s =$ the sum of series; derive the formula for $l =$ the last term.

9. If $\frac{\sqrt{a-bx} + \sqrt{c-mx}}{\sqrt{a-bx} + \sqrt{nx-d}} = \frac{\sqrt{a-bx} - \sqrt{c-mx}}{\sqrt{a-bx} - \sqrt{nx-d}}$, prove by using the principles of proportion that $\frac{c-mx}{nx-d} = 1$.

SEPTEMBER, 1885.

1. Reduce $\dfrac{\dfrac{a^2}{b^3}+\dfrac{1}{a}}{\dfrac{a}{b}-\dfrac{1}{b}+\dfrac{1}{a}}$ to a simple fraction.

2. Find the greatest common divisor of $x^4 - 6x^2 - 8x - 3$ and $4x^3 - 12x - 8$.

3. Given $\sqrt{13+x} + \sqrt{13-x} = 6$, to find x.

4. Given $x^4 - 21x^2 = 100$, to find four values for x.

5. Find the value of $a^{\frac{1}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{1}{3}}$ when $a = 8$ and $b = 64$.

6. Given $\left\{\begin{matrix} x + y = a \\ x^2 - y^2 = b^2 \end{matrix}\right\}$, to find x and y.

7. Given $(x^2 - ax) : \sqrt{x} :: \sqrt{x} : x$, to find values of x.

8. Expand $\dfrac{1}{(2a-3)^{\frac{1}{2}}}$ into a series.

9. Compute the value of the continued fraction

$$\cfrac{1}{12+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3}}}}$$

JUNE, 1886.

1. Divide $\dfrac{c-b}{c+b} - \dfrac{c^3-b^3}{c^3+b^3}$ by $\dfrac{c+b}{c-b} + \dfrac{c^2+b^2}{c^2-b^2}$.

2. Divide $x^2y^{-\frac{4}{3}} - 2 + x^{-2}y^{\frac{4}{3}}$ by $x^{\frac{1}{2}}y^{-\frac{1}{3}} - x^{-\frac{1}{2}}y^{\frac{1}{3}}$.

3. Multiply $\sqrt{-a} + c\sqrt[3]{b}$ by $\sqrt{-a} - c\sqrt[3]{b}$.

4. In $\dfrac{1}{\sqrt{3}-1}$ make the denominator rational, and compute the value of the expression to three places of decimals.

5. Given $a + x = \sqrt{a^2 + x\sqrt{b^2 + x^2}}$, to find x.

6. Solve the equations $\begin{cases} x + y = 12, \\ x^2 + y^2 = 74. \end{cases}$

7. If $A : B = C : D$, prove by the principles of proportion that $A^2 - B^2 : B^2 = C^2 - D^2 : D^2$.

8. Find the sum of the infinite series $\frac{1}{5} + \frac{1}{2}\frac{1}{5} + \frac{1}{1}\frac{1}{2}\frac{1}{5} +$ etc.

9. Expand to four terms by the Binomial Theorem $\dfrac{1}{\sqrt{1+x^2}}$.

SEPTEMBER, 1886.

1. Divide $\dfrac{x^4 - y^4}{x^2 - 2xy + y^2}$ by $\dfrac{x^2 + xy}{x - y}$.

2. Multiply $a^{\frac{5}{2}} - a^2 b^{\frac{1}{3}} + a^{\frac{3}{2}} b^{\frac{2}{3}} - ab + a^{\frac{1}{2}} b^{\frac{4}{3}} - b^{\frac{5}{3}}$ by $a^{\frac{1}{2}} + b^{\frac{1}{3}}$.

3. Free the fraction $\dfrac{1 - a^{-2} - y^2}{1 - x^{-8} y^{-2} + x^{-2}}$ from negative exponents.

4. Find x from $\dfrac{7x + 9}{4} - \left(x - \dfrac{2x - 1}{9}\right) = 7$.

5. Find x, y, and z from $\begin{cases} a = y + z, \\ b = x + z, \\ c = x + y. \end{cases}$

6. Multiply $x - 5 + 2\sqrt{-1}$ by $x - 5 - 2\sqrt{-1}$.

7. Make the denominator of the following fraction rational:

$$\frac{\sqrt{x} - \sqrt{x+y}}{\sqrt{x} + \sqrt{x+y}}.$$

8. Solve the equation $\frac{1}{x-1} + \frac{2}{x-2} = \frac{4}{3}$.

9. If $a : b = c : d$, prove by the principles of proportion that

$$\frac{a+b+c+d}{a+b-c-d} = \frac{a-b+c-d}{a-b-c+d}.$$

10. In a geometrical progression, having given first term, ratio, and sum of series, write formula for last term.

11. Expand to 4 terms $(a+x)^{-\frac{1}{4}}$.

JUNE, 1887.

1. Resolve each of the following expressions into three factors:

$$a^4 b + 8\,a\,c^3\,b\,m^6, \qquad 4\,c^3 x^2 + 4\,c^2 x\,y + c\,y^2.$$

2. Divide $\frac{a}{a-b} - \frac{b}{a+b}$ by $\frac{b}{a-b} - \frac{a}{a+b}$.

3. Multiply $\left(x - \frac{1-\sqrt{3}}{2\sqrt{2}}\right)\left(x - \frac{1+\sqrt{3}}{2\sqrt{2}}\right)\left(x + \frac{1}{\sqrt{2}}\right)$.

4. Solve $\sqrt{x} + 40 = 10 - \sqrt{x}$.

5. Solve $m x^2 + m n = 2 m \sqrt{n x} + n x^2$.

6. Given $\frac{15}{x} : \frac{21}{y} :: 3 : 7$, and $x^2 - y^2 = 9$, to find x and y

7. Expand by the Binomial Theorem $3\,b\,(2\,x - y)^{\frac{1}{2}}$.

SEPTEMBER, 1887.

1. Free the following from brackets, and combine the terms containing x and y.

$$a + b\{x - 3a(y - 2x) + 3x[4a + 2(4b + 3)]\}.$$

2. Factor $a^8 - 27b^3c^6$ and $m^2x^4 - 2m^2x^2y^2 + m^2y^4$.

3. Multiply $\frac{x^4 - b^4}{x^2 - 2bx + b^2}$ by $\frac{x^2 + bx}{x - b}$.

4. Multiply $a^{\frac{2}{3}} - a^{\frac{1}{3}} + 1 - a^{-\frac{1}{3}} + a^{-\frac{2}{3}}$ by $a^{\frac{1}{3}} + 1 + a^{-\frac{1}{3}}$.

5. Solve $\frac{x}{a} + \frac{y}{b} = 1.$

$$\frac{x}{a} + \frac{z}{c} = 1.$$

$$\frac{y}{b} + \frac{z}{c} = 1.$$

6. Solve the equation $4x^2 - \sqrt{3x} + 16 = 2x^2$.

7. Make the denominator rational in the fraction $\frac{1}{a^{\frac{1}{2}} - b^{\frac{1}{3}}}$.

8. Expand by the Binomial Theorem $\frac{1}{(a-b)^{\frac{3}{2}}}$.

JUNE, 1888.

1. Remove the parentheses from the following expression and reduce it to its simplest form.

$$5x - (3x - 4) - [7x + (2 - 9x)].$$

2. Resolve each of the following expressions into as many factors as possible.

(*a.*) $x^6 - 1.$

(*b.*) $(x^2 + y^2 - z^2)^2 - 4x^2y^2.$

3. Divide $\frac{1}{1-x} - \frac{1}{1+x}$ by $\frac{1}{1-x} + \frac{1}{1+x}$.

4. Solve the equations

$$\frac{3}{x}+\frac{1}{y}=\frac{5}{4}.$$

$$\frac{2}{x}-\frac{3}{y}=-1.$$

5. Solve the equation $\sqrt{x-3}-\sqrt{2x+8}=-3$.

6. Solve the equation $x^3-x^{\frac{3}{2}}=256$.

7. Multiply $x+3-2\sqrt{-1}$ by $x+3+2\sqrt{-1}$.

8. Expand $(x^3+b)^{-\frac{1}{3}}$ to four terms.

9. Given the series $y=x-\frac{x^2}{2}+\frac{x^3}{4}-\frac{x^4}{8}+$ etc., to find the value of x in terms of y.

JUNE, 1889.

1. Simplify

$$3x-4y+5\,[-4x-\{3y-(2+7x-2y)-4\}].$$

2. Solve $\frac{3}{x}-\frac{5}{y}+2=0$, $\frac{5}{x}-\frac{3}{y}-2=0$.

3. Simplify $\left(\frac{x^2+y^2}{x^2-y^2}-\frac{x^2-y^2}{x^2+y^2}\right)\div\left(\frac{x+y}{x-y}-\frac{x-y}{x+y}\right)$.

4. Give the four factors of the first degree in x and y of the expression $9x^4-40x^2y^2+16y^4$.

5. Solve $6x^2-5x=6$; solve also $x(1-x)=ax^2+b$.

6. Rationalize the denominator of $\frac{3-5\sqrt{-3}}{5+3\sqrt{-3}}$.

7. Expand $\frac{1}{\sqrt{a - x^2}}$ in ascending powers of x.

8. State and prove the formula expressing (s) the sum of n terms of a geometric progression having the first term a and the ratio r.

SEPTEMBER, 1889.

1. Simplify

$$-[13x - 14y - 3\{2x - 2(-y + 2x) - 7y\}].$$

2. Solve $\frac{2}{\sqrt{x}} - \frac{3}{\sqrt{y}} + 1 = 0, \quad \frac{3}{\sqrt{x}} - \frac{2}{\sqrt{y}} - 1 = 0.$

3. Simplify $\dfrac{x}{x-a} - \dfrac{x}{x+a} - \dfrac{\dfrac{x+a}{x-a} - \dfrac{x-a}{x+a}}{\dfrac{x+a}{x-a} + \dfrac{x-a}{x+a}}.$

4. Factor completely $(x - y)^7 - (x^7 - y^7)$.

5. Solve $12x^2 - 25x = -12.$

Solve also $(2 + x)(1 - x) = ax^2.$

6. Express as a fraction with a rational denominator

$$\frac{4 - 3\sqrt{-5}}{3 + 2\sqrt{-5}}.$$

7. Expand $(a - 2x)^{-\frac{2}{3}}$ in ascending powers of x.

8. What is the number of permutations of the letters of the word *muriate* taken three at a time? Prove that the number of permutations of any number of things taken t at a time is $1 \cdot 2 \cdot 3 \ldots t$ times the number of combinations of the things taken t at a time.

EXAMINATIONS FOR ADMISSION TO AMHERST COLLEGE.

JUNE, 1878.

1. Divide $\frac{a^4 - m^4}{a^2 - 2am + m^2}$ by $\frac{a^2 + am}{a - m}$.

2. Reduce $\frac{a^2 - 1}{ab - b}$ to its lowest terms.

3. Given $b - \frac{1 + x}{1 - x} = 0$; to find x.

4. $2x + 3y = 23$; $5x - 2y = 10$; find x and y.

5. Find the cube root of $a^3\sqrt[4]{b^2}$

6. Divide $4\sqrt{24ac}$ by $2\sqrt{8a}$.

7. A father's age is twice that of his son; but 10 years ago it was three times as great. What is the age of each?

8. If 1 be added to the numerator of a fraction, its value is $\frac{1}{3}$; and if 1 be added to the denominator, its value is $\frac{1}{4}$. What is the fraction?

SEPTEMBER, 1878.

1. Resolve $1 - 36y^2$ into two factors.

2. Find the least common multiple of $9a^3$, $12a^2x^3$, and $24ax^2y$.

3. Find the sum of $x + \frac{a}{b}$ and $\frac{-x + d}{m - y}$.

4. Divide $\frac{x}{x^2 - 1}$ by $\frac{x + 1}{x - 1}$.

5. Given $8a = \dfrac{1-x}{1+x}$, to find x.

6. Reduce $7\sqrt{9a^2 - 27a^2b}$ to its simplest form.

7. Find the square root of $4a^4 - 12a^3 + 5a^2 + 6a + 1$.

8. What numbers are those whose difference is 20 and the quotient of the greater divided by the less is 3?

JUNE, 1879.

1. Reduce $3a - (2a - [a + 2])$ to its simplest form.

2. Find the greatest common divisor of $2x^3 - 7x^2 + 5x - 6$ and $3x^3 - 7x^2 - 7x + 3$.

3. Reduce $\dfrac{\dfrac{1}{1-x} - \dfrac{1}{1+x}}{\dfrac{1}{1-x} + \dfrac{1}{1+x}}$ to its simplest form.

4. Resolve $a^6 - b^6$ into four factors.

5. Given $7x - \dfrac{11x - 3}{4} = 3x + 7$; find x.

6. A crew can row 20 miles in 2 hours down stream, and 12 miles in 3 hours against the stream. Required the rate per hour of the current, and the rate per hour of the crew in still water.

7. Extract the square root of $9x^4 - 12x^3 + 16x^2 - 8x + 4$.

8. Express $2x^2y^{\frac{1}{2}} - 3x^{-1}y^{\frac{3}{2}} - x^{-4}y^{-\frac{3}{2}}$ with positive exponents.

9. Divide x^{m-1} by $x^{\frac{mn-m}{n}}$ and reduce the quotient to its simplest form.

10. Multiply $a + b\sqrt{(-1)}$ by $a - b\sqrt{(-1)}$.

SEPTEMBER, 1879.

1. Interpret a^2; a^0; a^{-2}; $a^{\frac{3}{2}}$.

2. Multiply $a - b$ by $c - d$, and deduce the rule that "like signs give $+$ and unlike give $-$."

3. Separate into prime factors $3m^4x - 3n^4x$.

4. Explain the reason of the following equations:

$$a^m a^n = a^{m+n};\ a^m \div a^n = a^{m-n};\ (a^m)^n = a^{mn}.$$

5. $\dfrac{2x-9}{27} - \dfrac{x-3}{4} + \dfrac{x}{18} = \dfrac{25-3x}{3}$; find x.

6. A left a certain town at the rate of a miles an hour, and in n hours was followed by B at the rate of b miles an hour. In how many hours did B overtake A?

7. The sum of two numbers is s and the difference is d. What are the numbers? Show from the result that if from the greater of two numbers you subtract one half the sum, the remainder will be one half the difference.

8. Find the square root of $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$.

9. Simplify $7\sqrt[n]{(a^{2n}b^{2mn}c^p)}$.

10. Subtract $3\sqrt[6]{a^3}$ from $6\sqrt[4]{a^2}$.

JUNE, 1880.

1. Reduce $(a + b - c)\sqrt{x+y} - (a + b + c)(x+y)^{\frac{1}{2}}$ to its simplest form.

2. Resolve $a^6 - b^6$ into its prime factors.

3. Divide $\dfrac{2a^2x^{\frac{1}{2}}y}{6b^3c^{\frac{1}{4}}d^2}$ by $\dfrac{ax^{\frac{1}{2}}y^2}{b^2c^{\frac{5}{4}}d^2e}$.

4. $\frac{3x+2a}{2} - \frac{x-5a}{3} = 5a$; find x.

5. What number multiplied by m gives a product a less than n times the number?

6. $5x + 3y = 19$, $7x - 2y = 8$; find x and y.

7. Find the square root of $a + 2a^{\frac{1}{2}}x^{\frac{1}{2}} + x$.

8. Find the square root of $81a^4x^{-2}y^{\frac{2}{3}}z^{-\frac{1}{2}}$.

9. Free $\frac{a^{-2}+b^{-5}}{c^{-4}-d^{-7}}$ from negative exponents, and reduce the result to its simplest form.

10. Multiply $a + b\sqrt{-1}$ by $a - b\sqrt{-1}$.

SEPTEMBER, 1880.

1. Factor $81a^3 - 1$.

2. Find the greatest common divisor of

$$a^3 - a^2x + 3ax^2 - 3x^3, \text{ and } a^2 - 5ax + 4x^2.$$

3. Reduce $a + b - \frac{a^2 - b^2 - 5}{a - b}$ to a fractional form.

4. $\frac{x}{2} - 12 = \frac{y}{4} + 8$; $\frac{x+y}{5} + \frac{x}{3} - 8 = \frac{2y-x}{4} + 27$; find x and y.

5. Find the square root of $9x^4 - 12x^3 + 16x^2 - 8x + 4$.

6. Find the sum of $\sqrt{3a^2b}$ and $\sqrt{3x^2b}$.

7. Multiply $5a^{\frac{1}{2}}$ by $3a^{\frac{1}{2}}$.

8. Find the cube of $a - bx^{\frac{1}{2}}$.

9. Find the cube root of $(x + y)\sqrt{x + y}$.

10. Simplify $\dfrac{a + b\sqrt{-1}}{a - b\sqrt{-1}} + \dfrac{a - b\sqrt{-1}}{a + b\sqrt{-1}}$.

JUNE, 1881.

1. Remove the parentheses and reduce to its simplest form $a - \{a - [a - (a - x)]\}$.

2. Find the greatest common divisor of $8x^2 + 2x - 3$ and $12x^3 + 10x^2 - 4$.

3. Resolve $x^{12} - y^{12}$ into its simplest factors.

4. Reduce $\dfrac{x^2}{x^2 - 1} + \dfrac{x}{x + 1} - \dfrac{x}{1 - x}$ to its simplest form.

5. Find the square root of

$$9x^6 - 12x^3y^3 + 16x^2y^4 - 24x^4y^2 + 4y^6 + 16xy^5.$$

6. $y - a = 3(x - b)$; $y - b = 2(x - a)$; find x and y.

7. Multiply $\sqrt[3]{4a}$ by $\sqrt{6x}$.

8. A wine-merchant has two kinds of wine which cost 90 cents and 36 cents a quart, respectively. How much of each must he take to make a mixture of 100 quarts worth 50 cents a quart?

9. Divide $x^{\frac{3}{2}} + x^{\frac{1}{2}} - 6$ by $x^{\frac{1}{2}} - 2$.

10. Reduce $\left[\left(\dfrac{-a^2b^3}{c^3d}\right)^2\right]^3$ to its simplest form.

11. If $a : b = c : d$, prove $a + b : a - b = c + d : c - d$.

12. $3x^2 - 4x = 7$; find x.

13. Insert three geometrical means between 13 and 208.

14. Demonstrate the two fundamental formulæ of Arithmetical Progression.

15. What is the Binomial Theorem? Give the first four terms of $(a + x)^{10}$.

SEPTEMBER, 1881.

1. Reduce $1 - \frac{a}{a - x} - \frac{x^2}{a^2 - x^2}$ to a single fraction.

2. Find all the factors of $x^6 - 64\,a^6$.

3. $\frac{x}{a} + \frac{x}{b - a} = \frac{a}{a + b}$. Find x.

4. $\left\{\begin{array}{l} 7(x + y) + 3(x - y) = 80 \\ 7(x + y) - 3(x - y) = 32 \end{array}\right\}$. Find x and y.

5. Find the greatest common divisor of $x^3 + 3x^2 + 4x + 12$, and $x^3 + 4x^2 + 4x + 3$.

6. Find the square root of $4x^4 - 12x^3 + 5x^2 + 6x + 1$.

7. Find two numbers whose sum is a and whose difference is b.

8. Subtract $\sqrt[10]{a^{10}\,b^2}$ from $3\,a\,\sqrt[5]{b}$.

9. $2x = 4 + \frac{6}{x}$. Find x.

10. Insert two geometrical means between 24 and 192.

11. Expand $(2x + y)^5$ by the binomial theorem.

JUNE, 1882.

1. Reduce $\frac{x^{2n} - y^{2n}}{x^n + y^n}$ to its lowest terms.

2. Find a number of two digits such that it shall be equal to 7 times the sum of its digits, and if 27 be subtracted from the number, the digits will be inverted.

3. Reduce $4\sqrt{2} - 6\sqrt{8} + 10\sqrt{32}$ to its simplest form.

4. Extract the square root of $81\,a^4x^{-2}y^{\frac{1}{2}}z^{-\frac{1}{2}}$.

5. Solve the equation $3x^2 - 4x = 119$.

6. Form a quadratic equation whose roots shall be -3 and -2.

7. Solve the equations $\left\{\begin{matrix} y^2 + xy = 15 \\ x^2 + xy = 10 \end{matrix}\right\}$.

8. What proportions may be derived from the proportion

$$a : b = c : d\,?$$

9. Find the sum of the first n odd numbers

$$1 + 3 + 5 \ldots 2n - 1.$$

10. What is the sum of the infinite descending series

$$\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \text{etc.}\,?$$

11. Give the first four terms of $(1 + 2x^2)^n$.

SEPTEMBER, 1882.

1. Divide $m^3 - n^3$ by $m^2 + mn + n^2$.

2. Multiply $x^4 - x^3y + x^2y^2 - xy^3 + y^4$ by $x + y$.

3. Reduce $a - \frac{2ac - c^2}{a}$ to a fraction.

4. Given $\frac{x+a}{3} - \frac{3(x+a)}{4} = \frac{1}{3}(x+a) - a$, to find x.

5. Develop $\left(3x - \frac{1}{x}\right)^4$.

6. A courier starts from a place and travels a miles a day; n days after he is followed by another, who travels b miles a day; in what time will the second overtake the first?

7. Given $\left\{\begin{array}{l} x^2 + xy = 10 \\ xy + 2y^2 = 20 \end{array}\right\}$, to find x and y.

8. Divide $15\,(a^3b^5)^{\frac{1}{2}}$ by $3\,(ab^2)^{\frac{1}{2}}$.

9. Find the n^{th} term of the series 2, $2\frac{1}{3}$, $2\frac{2}{3}$, etc.

10. Find the sum of n terms of the series $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27}$, etc.

JUNE, 1883.

1. Find the value of

$$6a - [4b - \{4a - (6a - 4b)\}].$$

2. Divide $a^{-3n} - b^{6n}$ by $a^{-n} - b^{2n}$.

3. Show that $a^0 = 1$; also that $a^{-m} = 1 \div a^m$.

4. Resolve $a^{4m} - b^{4m}$ into its prime factors.

5. Find the greatest common divisor of

$$a^4 - b^4 \text{ and } a^3 + a^2b - ab^2 - b^3.$$

6. Given $3ax - 2bx - \frac{1}{3}c - \frac{1}{4}mx = \frac{2}{3}c + \frac{3}{4}mx - n - bx + 2ax$, to find x.

7. Divide the number a into two parts, such that the second part shall equal m times the first part plus n.

8. $\left\{\begin{matrix} 3y - 2x = 9 \\ 7x + y = 26 \end{matrix}\right\}$; find x and y.

9. Multiply $\sqrt{a+c}$ by $\sqrt[3]{a+c}$.

10. $3x^2 - 4x = 15$; find x.

11. Expand $(1+x^2)^7$ by the Binomial Formula.

12. Find the $(2n)$th term of the series 1, 3, 5, 7 . . .

SEPTEMBER, 1883.

1. Multiply $a^{m-1}b + a^m b^{-2} - a^{n-2}b^{-1}$ by ab.

2. Find the greatest common divisor of

$$3x^2 - 4xy + y^2, \text{ and } 5x^3 - 6x^2y + y^3.$$

3. $\left\{\begin{matrix} 2x - y = 9 \\ 2y - 3x = -11 \end{matrix}\right\}$; find x and y.

4. A boatman can row 6 miles an hour with the current, and 2 miles an hour against it; how far down the stream can he row so as to return to his place of starting in 12 hours?

5. Extract the square root of

$$1 + 4x^2 + 9x^4 + 4x + 6x^2 + 12x^3.$$

6. Find the square root of $64\sqrt[3]{36\,a^4b^{-2}}$.

7. $x^2 + \frac{1}{6}x = \frac{1}{6}$; find x.

8. Give the first three and the last three terms of $(x-y)^{100}$ with their proper signs.

9. Find the sum

$$\frac{n-1}{n} + \frac{n-2}{n} + \frac{n-3}{n} +, \text{ etc., to } n \text{ terms.}$$

10. Insert two geometrical means between $\frac{1}{9}^0$ and $\frac{1}{4}^5$.

JUNE, 1884.

1. Write the equivalent of $\frac{(x-y)^{-4}}{x^{-4}+y^{-4}}$ without negative exponents, and reduce the result to its simplest form.

2. Find the greatest common divisor of

$$x^3 - 2x^2y + xy^2 - 2y^3, \text{ and } 2x^4 - 4x^3y - xy^3 + 2y^4.$$

3. Find the sum of

$$\sqrt{9ab^3},\ -\sqrt{a^3b}, \text{ and } \sqrt{a^3b - 6a^2b^2 + 9ab^3}.$$

4. What fraction is that which becomes $\frac{2}{3}$ when its numerator is increased by 3, and equal to $\frac{1}{2}$ when its denominator is diminished by 1?

5. If $\frac{1}{2}a - x : \frac{1}{2}a + x = b - y : b + y$, show that $2x : y = a : b$.

6. Solve $$2x = 4 + \frac{6}{x}.$$

7. Given d the common difference, n the number of terms, and s the sum of an arithmetical progression, to find the last term l.

8. Insert five geometrical means between $\frac{1}{3}$ and $21\frac{1}{3}$.

9. Expand $(e^x - e^{-x})^4$ by the Binomial Formula.

10. Solve $x^2 - xy = -2$; $xy - y^2 = -3$.

SEPTEMBER, 1884.

1. Find the greatest common divisor of $x^3 - 2x^2 - x + 2$ and $x^4 - 3x^3 + 3x^2 - 3x + 2$.

2. The sum of the digits of a number of two figures is 9; and if 9 be subtracted from the number the digits are reversed. What is the number?

3. $\left.\begin{array}{l} ax + by = c \\ a'x + b'y = c' \end{array}\right\}$; find x and y.

4. The sum of the squares of two consecutive numbers is 545. What are the numbers?

5. Solve $\begin{cases} x - y = 2. \\ x^2 - y^2 = 20. \end{cases}$

6. Raise $2\sqrt[6]{3x^5y}$ to the 4th power and reduce the result to its simplest form.

7. What number added to 2, 20, 9, 34, will make the result proportional?

8. Given d, the difference, n, the number of terms, and s, the sum of an arithmetical progression; find the formula for l, the last term.

9. Find the sum of the infinite series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} +$ etc.

10. Find the 4th term of $(x - 2y)^{12}$ by the Binomial Theorem.

JUNE, 1885.

1. Divide $\frac{x-y}{x^2+2xy+y^2}$ by $\frac{x^2-y^2}{x+y}$.

2. Factor $x^4 - 2x^2y^2 + y^4$.

3. Reduce the product $(3x^{-2}y^3z^{-4})(5x^3y^{-4}z^3)$ to its simplest form, freed from negative exponents.

4. Solve $\sqrt{x+13} = 1 + \sqrt{x}$.

5. Solve $x^2 + 4ax = b$.

6. Given $\begin{cases} x^2 - xy = 15, \\ x^2 - y^2 = 21; \end{cases}$ find x and y.

7. Given $\frac{3x}{4} + \frac{x-4}{2} - \frac{x-10}{2} = x - 6$, to find x.

8. What is the sum of the first twenty odd numbers?

9. The first term of a geometrical progression is $\frac{1}{4}$, the ratio $\frac{1}{2}$, the number of terms 7. Find the sum.

10. Give the fourth term of $(2x - 3y)^{-8}$.

SEPTEMBER, 1885.

1. From $a - \frac{b-c}{2}$ take $\frac{a-b}{3} - c$.

2. Divide $a^6 - 1$ into its prime factors.

3. Reduce to its lowest terms $\frac{4x^3 - 5x^2 + x}{8x^2 - 6x + 1}$.

4. Solve $\begin{matrix} 3x - 4y = -6, \\ 10x + 2y = 26. \end{matrix}$

5. Solve $2x^2 - 5x + 2 = 0$.

6. Solve $\begin{aligned} x^2 + y^2 &= 34, \\ x + y &= 8. \end{aligned}$

7. What two numbers whose difference is d are to each other as a to b.

8. Insert 5 arithmetical means between 2 and -3.

9. What is the sum of the first 10 terms of the series 1, 2, 4, 8, etc.?

10. Expand $(x^2 - 2b)^6$.

JUNE, 1886.

1. From $\frac{a+b-2c}{3}$ take $\frac{2a-b+c}{4}$.

2. Reduce $\frac{3x^2y + 3xy^2}{3x^2 + 6xy + 3y^2}$ to its lowest terms.

3. Given $3x - \frac{x-4}{4} - 4 = \frac{5x+14}{3} - \frac{1}{12}$, to find x.

4. Solve $x + \sqrt{x+3} = 4x - 1$.

5. Given $2x - \frac{y+3}{4} = 8$, $4y - \frac{8-x}{3} = 24\frac{1}{2} - \frac{2y+1}{2}$; find x and y.

6. Find $\sqrt{24} + \sqrt{54} - \sqrt{6}$.

7. A person sets out from a certain place, and goes at the rate of 11 miles in 5 hours; and, 8 hours after, another person sets out from the same place, and goes after him at the rate of 13 miles in 3 hours. How far must the latter travel to overtake the former?

8. The 1st and 9th terms of an arithmetical progression are 5 and 22. Find the sum of 21 terms.

9. Find the 12th term of the geometrical progression

$$\sqrt{2},\ -2,\ +2\sqrt{2},\ -4,\ \text{etc.}$$

10. Find the first four terms of $(2x - 3y)^5$ by the Binomial Formula.

SEPTEMBER, 1886.

1. Reduce $(a + b - c)^2 + (a - b + c)^2$ to its simplest form.

2. Reduce $\dfrac{2x^3 - 16x - 6}{3x^3 - 24x - 9}$ to its lowest terms.

3. Given $\dfrac{3x + 4}{5} - \dfrac{7x - 3}{2} = \dfrac{x - 16}{4}$; find x.

4. Find x and y from the equations

$$\frac{x - 2}{5} - \frac{10 - x}{3} = \frac{y - 10}{4},$$

$$\frac{2y + 4}{3} - \frac{2x + y}{8} = \frac{x + 13}{4}.$$

5. Solve $2x - \sqrt{2x - 1} = x + 2$.

6. Solve $\begin{cases} x^2 + y^2 = 50, \\ 9x + 7y = 70. \end{cases}$

7. Reduce $\sqrt{45c^3} - \sqrt{80c^3} + \sqrt{5a^2c}$ to its simplest form.

8. Find the sum of the first 90 odd numbers by arithmetical progression.

9. Find the sum of the geometrical progression 20, 19, $18\frac{1}{20}$, etc.

10. Find the first four terms of $(1 - x)^{12}$ by the Binomial Formula.

JUNE, 1887.

1. Reduce $(a+b-c)\sqrt{x+y}-(a+b+c)(x+y)^{\frac{1}{2}}$ to its simplest form.

2. Resolve a^6-b^6 into its prime factors.

3. Divide $\dfrac{2a^2x^{\frac{4}{3}}y}{6b^3c^{\frac{1}{4}}d^2}$ by $\dfrac{ax^{\frac{1}{3}}y^2}{b^2c^{\frac{1}{4}}d^2e}$

4. $\dfrac{3x+2a}{2}-\dfrac{x-5a}{3}=5a$; find x.

5. What number multiplied by m gives a product a less than n times the number?

6. $\begin{matrix}5x+3y=19\\7x-2y=18\end{matrix}$; find x and y.

7. Find the square root of $a+2a^{\frac{1}{2}}x^{\frac{1}{2}}+x$.

8. Find the square root of $81a^4x^{-2}y^{\frac{2}{3}}z^{-\frac{1}{5}}$.

9. Find the roots of $ax^2+bx+c=0$.

10. $x^2+xy=10$, $xy-y^2=-3$; find x and y.

SEPTEMBER, 1887.

1. Reduce $a-[2b-(3c+2b-a)]$ to its simplest form.

2. Divide a^nb^{m-n} by $a^{n-m}d^{-n}$.

3. Reduce $\dfrac{\dfrac{c}{c-1}-1}{1-\dfrac{c}{c+1}}$ to its simplest form.

4. Given $\left\{\begin{matrix}x+2y=7\\2x+3y=12\end{matrix}\right\}$, to find x and y.

5. Reduce $a\sqrt{48\,a^3\,d}$ and $\sqrt{\frac{4}{49}}$ to simpler forms.

6. Multiply $3\sqrt{\frac{a}{3}}$ by $2\sqrt{\frac{a}{6}}$.

7. Given $\frac{x-5}{3}+\frac{x}{2}=12-\frac{x-10}{3}$, to find x.

8. Find two numbers whose sum equals s, and whose difference equals d.

9. Solve the equation $3x^2-4x=119$.

10. Find the first four terms of $(x-2y)^7$.

JUNE, 1888.

1. Resolve $16\,a^6b^4m^2-8\,a^3b^2m+1$ into its factors.

2. Find the greatest common divisor of $6x^3-6x^2y+2xy^2-2y^3$ and $12x^2-15xy+3y^2$.

3. $\frac{x+3}{2}-\frac{x-2}{3}=\frac{3x-5}{12}+\frac{1}{4}$; find x.

4. $\frac{x+y}{2}-\frac{x-y}{3}=8$; $\frac{x+y}{3}+\frac{x-y}{4}=11$; find x and y.

5. $x-2y+3z=2$; $2x-3y+z=1$; $3x-y+2z=9$; find x and y.

6. Solve the equations $\sqrt{x+5}=\frac{12}{\sqrt{x+12}}$.

7. Solve the equations $x+y=a$; $x^2+y^2=b^2$.

8. Find the sum $\frac{3}{5}\sqrt{\frac{2}{3}}$ and $\frac{3}{4}\sqrt{\frac{2.5}{6}}$.

9. Demonstrate the fundamental formulæ used in Arithmetical Progression. Find the sum of the first n terms of the progression 1, 3, 5, 7, etc.

10. Find the sum of the first n terms of a geometrical progression whose first term is a, and third term c.

SEPTEMBER, 1888.

1. Find the value of $\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}$ when $x=\frac{4ab}{a+b}$.

2. Resolve $1-c^4$ into its prime factors.

3. Multiply together $\frac{1-x^2}{1+y}$, $\frac{1-y^2}{x+x^2}$, and $1+\frac{x}{1-x}$.

4. Solve the equation $\frac{x+3}{x-3}-\frac{x-3}{x+3}=a$.

5. $\frac{x}{5}+\frac{y}{6}=18$; $\frac{x}{2}-\frac{y}{4}=21$; find x and y.

6. $x^2+y^2=34$; $xy=15$; find x and y.

7. Reduce $\frac{x}{a-\sqrt{a^2-x^2}}$ to an equivalent fraction having a rational denominator.

8. Find the ratio of an infinite decreasing geometrical progression of which the first term is 1, and the sum of the terms is $\frac{5}{4}$.

9. Find the sum of the terms of an arithmetical progression formed by inserting 9 arithmetical means between 9 and 109.

10. Expand $(a-b)^6$ by the Binomial Formula.

JUNE, 1889.

1. Find the greatest common divisor of

$$x^3 - 2x^2 - x + 2 \text{ and } x^4 - 3x^3 + 3x^2 - 3x + 2.$$

2. Multiply $a^m - a^n$ by $2a - a^n$.

3. $\left.\begin{array}{l} ax + by = c \\ a'x + b'y = c' \end{array}\right\}$; find x and y.

4. The sum of the squares of two consecutive numbers is 545. What are the numbers?

5. $\left.\begin{array}{l} x - y = 2 \\ x^2 - y^2 = 20 \end{array}\right\}$; find x and y.

6. Multiply $4a\sqrt{\frac{3a^2}{8b}}$ by $\frac{1}{2}a\sqrt{\frac{3}{2b}}$.

7. What number added to 2, 20, 9, 34, will make the sums proportional?

8. Given d the difference, n the number of terms, and s the sum of an arithmetical progression; prove the formula for l, the last term.

9. Find the sum of the infinite series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} +$ etc.

10. Find the fourth term of $(x - 3y)^{12}$ by the Binomial Theorem.

SEPTEMBER, 1889.

1. Factor $c^2 - (a^2 - 2ab + b^2)$; also $x^2 - 5x + 6$.

2. Free $\frac{x^2y^{-2}}{a^{-2}bx^{-1}y}$ from negative exponents.

3. Simplify $\frac{y - x + \frac{a}{2}}{7\frac{3}{4}}$.

4. Find x from the equation $\frac{7x+9}{4} - \left(x - \frac{2x-1}{9}\right) = 7$.

5. What are the numbers whose difference is 3, and the difference of whose squares is 51?

6. Solve the equations $2x - 5y = 9$, $3x + 2y = 23$.
7. What is the cube root of $-8a^{-3}b^{6}x^{-2}$?
8. Find the square root of $4x^2 - 12xy + 9y^2$.
9. Solve the equation $ax^2 + 2bx = c$.
10. Given $x^2 + y^2 = 130$; $x^2 - y^2 = 32$; find x and y.
11. Find the sum of the series, $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} +$ etc.

ENTRANCE EXAMINATIONS TO DARTMOUTH COLLEGE.

1878.

1. Define term, factor, coefficient, exponent, power, root, equation. What is the degree of a term? When is a polynomial homogeneous?

2. Write the following without using the radical sign:

$$\sqrt{a}; \qquad \sqrt[3]{a^2}; \qquad \sqrt{a^2+b^2-2ab}.$$

3. Write the following without using negative exponents:

$$a^{-2}; \qquad ab^{-1}; \qquad \frac{a^{-2}}{b^{-2}}.$$

4. Multiply $a-b\sqrt{-1}$ by $a+b\sqrt{-1}$. Also $a-b\sqrt{-1}$ by $a+c\sqrt{-1}$.

5. Raise $a-b\sqrt{-1}$ to the 3d power. Simplify the radical $(a^3-2a^2b+ab^2)^{\frac{1}{2}}$.

6. Solve $\frac{a^2-x^2}{a+x}-\frac{a^2-x^2}{a-x}=b$. Also $\frac{a}{x^{-1}}+bx^0+c=0$. Also $\frac{x-1}{2}-\frac{x-2}{3}=\frac{x+1}{6}$. Also $\frac{a^{\frac{1}{2}}-(a-x)^{\frac{1}{2}}}{a^{\frac{1}{2}}+(a-x)^{\frac{1}{2}}}=\frac{1}{a}$.

1880.

1. Define Algebra, factor, coefficient, exponent, fraction, equation.

2. Write the following without using the radical sign:

$$\sqrt{x^4}; \qquad \sqrt[4]{x^2}; \qquad \sqrt{a^2+b^2}; \qquad \sqrt{a^2+1+2a}.$$

3. Write the following without using negative exponents:

$$x^{-1}; \qquad \frac{a^{-1}}{b^{-1}}; \qquad (c^{-1})^{-1}.$$

4. Write, in the simplest form, the values of

$$\sqrt{\tfrac{3}{4}}; \qquad \sqrt{\tfrac{1}{4}}; \qquad \sqrt{4a^3}; \qquad 8^0; \qquad 8^{\frac{2}{3}}; \qquad 8^{-\frac{4}{3}}.$$

5. Find the product of $\sqrt{ab} \times -a^{\frac{1}{2}}b^{\frac{1}{2}} \times (-a^{\frac{1}{2}}b^{\frac{1}{2}}) \times 2ab$; also of $(2+\sqrt{-1})(2-\sqrt{-1})$.

6. Solve

$$\frac{x}{a+1} - b = \frac{x}{a-1}; \text{ also } \frac{x-2}{5} + \frac{.301}{.5} = .001x + .6 - \frac{x-2}{.05}.$$

1881.

1. Define term, factor, coefficient, exponent, power, root, equation.

2. What is the degree of a term?

When is a polynomial homogeneous?

3. Write the following without using the radical sign:

$$\sqrt[4]{c}; \qquad \sqrt[5]{a^2}; \qquad \sqrt{a^2+b^2-2ab}.$$

4. Write the following without using negative exponents:

$$x^{-2}; \qquad ax^{-1}; \qquad \frac{a^{-1}}{b^{-1}}.$$

5. Multiply $a - c\sqrt{-1}$ by $a + b\sqrt{-1}$.

6. Raise $b - a\sqrt{-1}$ to the 3d power.

Simplify the radical $\sqrt{c^3 - 2c^2b + cb^2}$.

7. Solve $\frac{a^2 - y^2}{a + y} - \frac{a^2 - y^2}{a - y} = a.$

8. Solve $\frac{x - 1}{2} - \frac{x - 2}{3} = \frac{x + 1}{6}.$

9. Solve $\frac{a}{x^{-1}} - b x^0 + c = 0.$

1882.

1. Define positive and negative quantities, coefficient, exponent, similar terms, elimination, radical quantity.

2. Resolve $x^4 - y^4$, $a^8 + 1$, and $1 + x^6$ into their prime factors.

3. Solve $\frac{x + 1}{x - 1} - 1 = \frac{a}{c}$; also $\frac{1}{1 - x} - \frac{1}{1 + x} = \frac{2x}{1 - x^2}$; also $\left\{\begin{matrix} ax + by = c \\ cx + dy = e \end{matrix}\right\}$.

4. Write the following without using the radical sign or negative exponents: $\sqrt[3]{a^4}$, $\sqrt{1 + 2a + a^2}$, $a b^{-1} c a^{-1}$, $\sqrt{a^{-2}}$.

5. Divide $a^2 + a(b + c)\sqrt{-1} - bc$ by $a + b\sqrt{-1}$.

6. Rationalize the denominators of $\frac{a}{\sqrt[3]{c}}$, $\frac{x}{\sqrt{c} - \sqrt{a}}$, $\frac{1}{1 - \sqrt{3}}$.

1883.

1. Divide $a^2 + \frac{1}{a^2} - 2$ by $\frac{1}{a} - a.$

2. Resolve $a^{12} - x^{12}$ into six factors.

3. Find the least common multiple of

$$x^3 - x,\ x^2 - x - 2, \text{ and } x^3 + 1.$$

4. Reduce

$$\left(\frac{x - 1}{x^2 + y^2} - \frac{y^2 x - x^3}{y^4 - x^4}\right)(y^2 + x^2)$$ to its simplest form.

5. Solve the equations:

$$(1)\quad 2x - \left(x - \frac{x-1}{3}\right) = \frac{5x}{4};$$

$$(2)\quad \begin{cases} \frac{x}{2} + \frac{y}{4} = 3 \\ \frac{2x}{3} - \frac{y}{6} = 2; \end{cases} \qquad (3)\quad \begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{1}{4} \\ \frac{1}{x} - \frac{1}{y} = \frac{1}{12}. \end{cases}$$

6. Write the values of $8^{-\frac{2}{3}}$, 8^0, $16^{\frac{3}{4}}$, and $(8\,a^{\frac{1}{2}}b^2)^{\frac{2}{3}}$.

7. Multiply together $\sqrt[3]{a\,b\,c^2}$, $a^{\frac{1}{3}}b^{-1}c^{\frac{1}{3}}$, and $a^{-\frac{2}{3}}b^{\frac{2}{3}}c^{-1}$.

8. Divide $a^3 - b^2$ by $a^{\frac{3}{2}} - \sqrt{b}$.

1884.

1. Resolve the following expressions into factors:

$4\,a^2 - 9\,b^4$; $a^6 + 1$; $9\,a^4 - 24\,a^2b^2 + 16\,b^4$; $x^2 + 2x - 3$.

2. Reduce $\left(\frac{1-x^3}{x-1} + \frac{1+x^3}{x+1}\right)\frac{1}{x}$ to its simplest form.

3. Solve the equations

(a) $3x - \left(3x - \frac{2-x}{4}\right) = \frac{x-2}{5}$;

(b) $\begin{cases} x + 2y = 6 \\ y - \frac{x}{2} = 1; \end{cases}$ (c) $\begin{cases} x + y = 5xy \\ \frac{1}{x} + \frac{2}{y} = 8; \end{cases}$

(d) $\sqrt{x+1} + \sqrt{x-1} = \frac{2}{\sqrt{x+1}}$.

4. Find the value of $16^{\frac{1}{4}} \times 5^0 \times 2^{-2} \times 8^{-\frac{1}{3}}[(-2)^2]^{\frac{1}{2}}$.

5. Simplify the following expressions:

(a) $(1 + \sqrt{-1})^2$; (c) $\frac{1}{\sqrt{x+1}}\sqrt{1-x^2}$;

(b) $(1 + \sqrt{-3})^3$; (d) $\sqrt{a^2 + \frac{1}{a^2} + 2}$.

6. Divide $x^2 + y^4$ by $x^{\frac{1}{2}} + y^{\frac{1}{4}}$.

1885.

1. Factor $x^4 - 16\,a^4$, $x^6 + a^6$, $x^2 - 2\,a\,x - b^2 + a^2$, $x^2 - x - 72$.

2. Find the greatest common divisor and least common multiple of $x^3 - 3\,x - 2$ and $x^5 - 2\,x^4 - x + 2$.

3. Simplify $\dfrac{1 - \dfrac{2 + \frac{1}{x}}{2 - \frac{1}{x}}}{1 - \dfrac{x + \frac{1}{2}}{x - \frac{1}{2}}}$ and $\dfrac{8}{\sqrt{-3} - 1} - (1 - \sqrt{-3})^2$.

4. Solve $x - \left(\dfrac{1 - x}{4} - \dfrac{1 + x}{2}\right) = \dfrac{x + 3}{2}$.

5. Solve $\begin{cases} \frac{x}{2} + 2\,y = x - y + 4. \\ 2\,x - 3\,y = y - x + 4. \end{cases}$

6 Solve $\sqrt{x + 2} - \sqrt{x - 2} = \sqrt{2\,x}$.

7. Find the value of $9\frac{1}{2} \times 8^{-\frac{4}{3}} \times 7^0 \times 4^{-\frac{1}{2}} \div (8^{-2} \times 3)^{-1}$.

8. Multiply $x - x^{-1}$ by $x - x^{-1}$, $2 + \sqrt{-3}$ by $2\sqrt{3}$, and $x^{\frac{1}{3}} - y^{\frac{2}{3}}$ by $x^{\frac{2}{3}} + x^{\frac{1}{3}}y^{\frac{2}{3}} + y^{\frac{4}{3}}$.

1886.

1. Factor $x^2 - 9\,a^4$, $x^9 + y^9$, $x^2 + 4\,x\,y - 4 + 4\,y^2$, $x^2 + 3\,a\,x + 2\,a^2$.

2. Reduce $\left\{\dfrac{\dfrac{x^6 + 1}{x^2 + 1}}{\dfrac{x^3 + 1}{x + 1}}\right\}[1 + x\,(x - 1)]$ to its simplest form.

3. Multiply $x + x^{-1} - 1$ by $x - x^{-1} + 1$, and $x^{\frac{1}{2}} + y^{\frac{1}{2}}$ by $x^{\frac{3}{2}} - x\,y^{\frac{1}{2}} + x^{\frac{1}{2}}\,y - y^{\frac{3}{2}}$.

4. Divide $x - y$ by $x^{\frac{1}{4}} + y^{\frac{1}{4}}$, and $x^3 + x^{-3}$ by $x + x^{-1}$.

5. Write the value of $8^{\frac{1}{3}} \times 9^{-\frac{1}{2}} \times 2^{-1} \times 3^0 \times 1^{-2} \times \sqrt[4]{81}$.

6. Solve $x - 3 - \left(\frac{x-4}{4} - \frac{x-6}{7}\right) = \frac{x}{1\frac{2}{5}}$.

7. Solve $\begin{cases} \frac{x}{4} - \frac{y}{3} = x - 4 \\ \frac{x}{8} - y = y - 5\frac{1}{2} \end{cases}$ and $\begin{cases} \frac{1}{x} + \frac{1}{y} = 5. \\ \frac{2}{x} + \frac{3}{y} = 13. \end{cases}$

1887. (See Preface.)

1. Give all the theorems used in factoring binomials.

2. Find the prime factors of $1 + a^6$, $a^6 - b^3$, $a^4 + 4\,b^4 + 4\,a^2\,b^2 - c^8$, $x^2 - x - 20$, $x^3 + x^2 - 8\,x - 12$.

3. Find the G. C. D. of $x^4 + 4\,x^3 + 12\,x^2 + 16\,x + 16$ and $4\,x^3 + 12\,x^2 + 24\,x + 16$.

4. Solve $\frac{1}{y} + \frac{1}{x} = 7$, $\frac{2}{y} - \frac{3}{x} = 2$.

5. Write the values of $27^{-\frac{2}{3}}$, $27^{\frac{4}{3}}$, 27^0, $\left[(27^{-2})^{-\frac{1}{3}}\right]^{-\frac{1}{2}}$.

6. Reduce to simplest form

$$\frac{a^{-1} - b^{-1}}{(a-b)^{-1}} \times a\,b, \qquad \sqrt{x^2 + \frac{1}{x^2} - 2},$$

$$(11 + 4\sqrt{6})^{\frac{1}{2}}, \qquad (-1 - \sqrt{-3})^3.$$

7. Reduce to equivalent fractions having rational denominators

$$\frac{a\,c}{3\,a^{\frac{1}{2}}\,b^{\frac{3}{4}}\,c^{-\frac{1}{3}}}, \qquad \frac{1}{a^{\frac{2}{3}} + b^{\frac{1}{2}}}.$$

8. Solve $\sqrt{x+5} + \sqrt{x-8} = \sqrt{3}$.

1888. (See Preface.)

1. Remove the parentheses from

$$3a - \{3a - [3a - (3a - \overline{3a - 3a}) - 3a] - 3a\} - 3a,$$

and simplify the result.

2. Give the three theorems used in factoring binomials.

3. Factor $4\,a^2x^4 - 9\,b^4c^6$, $8\,b^2c^3 + 48\,b^2c^2 + 72\,b^2c$, $x^4 - 3\,x^3 - 14\,x^2 + 48\,x - 32$.

4. Resolve $1 - a^3$ into six factors.

5. Give two methods of finding the G. C. D. of two quantities.

6. Reduce $\dfrac{\dfrac{a^2+b^2}{b} - a}{\dfrac{1}{a} - \dfrac{1}{b}} \times \dfrac{a^2 - b^2}{a^3 + b^3}$ to a simple fraction and lowest terms.

7. Solve $\begin{cases} 2\,x^{-1} - \ \ y^{-1} = 22\,x^0, \\ 3\,x^{-1} - 3\,y^{-1} = 18\,y^0. \end{cases}$

8. A and B can do $\frac{1}{4}$ of a piece of work in 2 days. B can do $\frac{1}{3}$ of it in 6 days. How long would it take A to do $\frac{1}{3}$ of it?

9. Reduce to simplest form

$$\sqrt{\frac{a^2+b^2}{4}},\ \sqrt{6+\sqrt{-13}} \times \sqrt{6-\sqrt{-13}},\ (35-12\sqrt{6})^{\frac{1}{2}}.$$

10. Reduce to equivalent fractions having rational denominators

$$\frac{ac}{c^{\frac{4}{3}}\left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right)},\qquad \frac{1}{a^{\frac{2}{3}} + b^{\frac{3}{4}}}.$$

11. Solve $\dfrac{1}{x + \sqrt{x^2-1}} + \dfrac{1}{x - \sqrt{x^2-1}} = 12.$

1889.

1. Find the prime factors of

$$a^9 - b^9, \quad a^6 + b^6, \quad a^6 + b^6 + 2a^3b^3 - x^2 - 4y^2 + 4xy,$$

and

$$x^3 - 8x^2 + 13x - 6.$$

2. Find the greatest common divisor of $x^3 - x^2 - 8x + 12$ and $3x^2 - 2x - 8$.

3. Simplify the following expressions:

(a) $\dfrac{a^{\frac{1}{2}}}{a^{-\frac{1}{2}}}$, (b) $\dfrac{x^2 + x^{-2} - 2}{x - \frac{1}{x}}$, (c) $\dfrac{x^{\frac{3}{4}} - y^{\frac{3}{4}}}{x^{\frac{1}{4}} - y^{\frac{1}{4}}}$,

(d) $\sqrt[3]{8\sqrt{a^6 + b\,a^{12}}}$, (e) $\dfrac{4^{\frac{3}{4}} \times 3^{\frac{1}{2}} \times 2^{\frac{1}{3}}}{2^{\frac{3}{4}} \times 9^{\frac{1}{4}} \times 4^{\frac{1}{4}}}$,

(f) $\sqrt{14 + 4\sqrt{6}}$.

4. Solve the following equations:

(1) $\dfrac{x}{2} - \left(\dfrac{x-1}{3} - \dfrac{x-9}{5}\right) = x - 4x^0$;

(2) $\begin{cases} \dfrac{1}{x} - \dfrac{3}{y} = 3, \\ \dfrac{2}{x} - \dfrac{8}{y} = 4; \end{cases}$

(3) $2\sqrt{x-1} + 4\sqrt{4x-4} = 5\sqrt{x}$.

EXAMINATIONS FOR ADMISSION TO BROWN UNIVERSITY.

1878.

1. Reduce $\dfrac{\dfrac{c}{c-1}-1}{1-\dfrac{c}{c+1}}$ to a simple fraction.

2. Divide a into two parts, such that m times one shall be n times the other.

3. If 4 be subtracted from both terms of a fraction, the value will be $\frac{1}{3}$; and if 5 be added to both terms, the value will be $\frac{5}{6}$. What is the fraction?

4. Given $\sqrt{x-9}+\sqrt{x+11}=10$, to find x.

5. Given $2x+\dfrac{3x-6}{2}=5x-\dfrac{3x-3}{x-3}$, to find x.

1879.

1. Add $\dfrac{1}{x+3}$, $\dfrac{x+1}{x^2-3x+9}$, and $\dfrac{x^2+x+1}{x^3+27}$.

2. Multiply $(a+b)^{\frac{1}{2}}$ by $(a+b)^{\frac{3}{2}}$, and

Divide $(a+b)\sqrt{a^2-1}$ by $(a-b)\sqrt{a^2+2a+1}$, giving answers in simplest forms.

3. Given $\dfrac{a^2(a+c)}{(a-b)(x-a)}-\dfrac{a^2(b+c)}{(a-b)(x-b)}=\dfrac{(a^3+a^2x)n}{x^2-a^2}$ to find x.

4. Given $\sqrt{2+x}+\sqrt{x}=\dfrac{4}{\sqrt{2+x}}$, to find x.

5. Divide the number s into two such parts, that if m^2 be divided by the second, and this quotient multiplied by the first, the product is the same as if n^2 be divided by the first and the quotient multiplied by the second.

1880.

1. Find the H. C. D. of

$$6x^3 - 8yx^2 + 2y^2x \text{ and } 12x^2 - 15xy + 3y^2.$$

2. Given $ax + by = c$ and $mx = ny + d$, to find x and y.

3. Extract the cube root of

$$-99x^3 - 9x^5 + x^6 + 64 - 144x + 156x^2 + 39x^4.$$

4. Given $\dfrac{3\sqrt{x} - 4}{2 + \sqrt{x}} = \dfrac{15 + \sqrt{9x}}{40 + \sqrt{x}}$, to find x.

5. Two pipes, A and B, will fill a cistern in 70 minutes, A and C in 84 minutes, and B and C in 140 minutes. How long will it take each to fill it alone?

6. Given $\sqrt{5 + x} + \sqrt{x} = \dfrac{15}{\sqrt{5 + x}}$, to find x.

7. A gentleman bought two pieces of silk which together measured 36 yards. Each cost as many shillings per yard as there were yards in the piece, and the cost of the pieces were to each other as 4 to 1. Required the number of yards in each piece.

8. Given $x + \sqrt{5x + 10} = 8$, to find x.

9. Given $x^3 - x^{\frac{3}{2}} = 56$, to find x.

10. Given $x^2 + xy = 15$ and $xy - y^2 = 2$, to find x and y.

1881.

1. $\frac{x}{a}+\frac{y}{b}=1;\ \frac{x}{a}+\frac{z}{c}=1;\ \frac{y}{b}+\frac{z}{c}=1.$

2. There is a number consisting of two digits; the number is equal to seven times the sum of its digits; if 27 be subtracted from the number the digits interchange their places; find the number.

3. Reduce to their simplest forms $\sqrt[3]{\frac{4}{7}}$ and $\sqrt[5]{250\,x^3y^6z^8}$.

4. $\sqrt{x-3}-\sqrt{x-14}-\sqrt{4x-155}=0.$

5. $\frac{7}{x^2-4}-\frac{3}{x+2}=\frac{22}{5}.$

6. $x+y=3;\ x^2+y^2=29.$

7. Develop by binomial formula $(2a-3b)^5$.

8. In an arithmetical progression given the last term -47, the common difference -1, and the sum of the terms -1118; find the first term and the number of terms.

1882.

1. Rationalize denominator $\frac{3\sqrt{5}-2\sqrt{2}}{2\sqrt{5}-\sqrt{18}}$.

2. At what time between 8 and 9 o'clock is the hour hand of a watch 20 minute-spaces in advance of the minute hand?

3. Extract the cube root of $\frac{a}{3}\sqrt{\frac{a}{3}}$, and express the result in the simplest form.

4. Multiply $-6\sqrt{-3}$ by $-2\sqrt{-2}$.

5. $7(x+7)-\frac{7(3x+50)}{x}=0.$ Find the value of x.

6. $\left\{\begin{matrix}6x^2-5xy+2y^2=12\\3x^2+2xy-3y^2=-3\end{matrix}\right\}$. Find x and y.

7. Develop by Binomial Formula $(\frac{1}{2}a-\frac{2}{3}b^2)^4$.

8. In a Geometrical Progression, given the number of terms 8, the ratio $\frac{1}{2}$, and the sum of the terms $7\frac{1}{16}$, to find the first term and the last term.

1883.

1. Factor $x^4 - y^4$; also factor $4a^4 - 8a^3x + 4a^2x^2$.

2. $\dfrac{6x+7}{15} - \dfrac{2x-2}{7x-6} = \dfrac{2x+1}{5}$. Find the value of x.

3. A sum of money is divided equally among a certain number of persons; if there had been four more, each would have received a dollar less than he did; if there had been five fewer, each would have received two dollars more than he did. Find the number of persons and what each received.

4. Multiply

$$a^{\frac{3}{2}} - a^{\frac{1}{2}} + 1 - a^{-\frac{1}{2}} + a^{-\frac{3}{2}} \text{ by } a^{\frac{1}{2}} + 1 - a^{-\frac{1}{2}}.$$

5. $\sqrt{a+x} + \sqrt{a-x} = \sqrt{b}$. Find the value of x.

6. $x + y = 4$; $\dfrac{1}{x} + \dfrac{1}{y} = 1$. Find the values of x and y.

7. A boat's crew row $3\frac{1}{2}$ miles down a river and back again in 1 hour and 40 minutes; supposing the river to have a current of 2 miles per hour, find the rate at which the crew would row in still water.

8. Find the sum of six terms of the Geometrical Progression of which $\frac{8}{5}$ is the first term and $\frac{8}{3}$ the second term.

1884.

1. $$\begin{cases} x - \left\{ \dfrac{2y-3}{4} + \dfrac{3x-5}{6} \right\} = 2 - \dfrac{2x-3y-1}{12}; \\ \left\{ \dfrac{2x-y}{4} - \dfrac{3y-2}{3} \right\} + y = 1\frac{1}{6} - \dfrac{3-2x}{6}. \end{cases}$$

Find the values of x and y.

2. A and B run a mile. First A gives B a start of 44 yards and beats him by 51 seconds; at the second heat A gives B a start of 1 minute 15 seconds, and is beaten by 88 yards. Find the times in which A and B can run a mile separately.

3. Extract the square root of

$$25x^4 - 30ax^3 + 49a^2x^2 - 24a^3x + 16a^4.$$

4. Simplify $\dfrac{\{(a^m)^{\frac{1}{r}}(a^q)^{\frac{1}{n}}\}^{nr}}{\{\sqrt[q]{b^n}(\sqrt[m]{b})^r\}^{mq}} \div \left\{\left(\dfrac{a}{b}\right)^q\right\}^r$.

5. $3x^2 - 4x - 4 = 0$. Find values of x.

1. What are eggs a dozen when two more in a shilling's worth lowers the price one penny per dozen?

7. Develop by Binomial Formula $(\frac{1}{2}a^2 - \frac{2}{3}b)^5$.

8. Sum to 20 terms 2, 6, 10, 14, . . .

9. Sum to 6 terms $3 + 2 + \frac{4}{3} + \ldots$

10. Find the sixth term of 3, 6, 12, . . .

JUNE, 1885.

1. Find highest common divisor of $15a^2x^3 - 20a^2x^2 - 65a^2x - 30a^2$ and $12bx^3 + 20bx^2 - 16bx - 16b$.

2. $\dfrac{k}{z} + \dfrac{z}{k} + \dfrac{k(z-k)}{z(z+k)} - \dfrac{z(z+k)}{k(z-k)} = \dfrac{kz}{k^2 - z^2} - 2.$

3. A number is compounded of three figures whose sum is 17. The figure of the hundreds is double that of the units. When 396 is subtracted the order of the figures is reversed. What is the number?

4. Multiply $2\sqrt{-3} - 3\sqrt{-2}$ and $4\sqrt{-3} + 6\sqrt{-2}$.

5. Reduce to an equivalent fraction with a rational denominator

$$\frac{\sqrt{x} - 4\sqrt{x-2}}{2\sqrt{x} + 3\sqrt{x-2}}.$$

6. $\sqrt{2x-3} - \sqrt{8x+1} + \sqrt{18x-92} = 0$. Find value of x.

7. $2x^2 - 2xy - y^2 = 3$; $x^2 + 3xy + y^2 = 11$. Find values of x and y.

8. In an arithmetical progression, given the last term, -47; the common difference, -1; and the sum of the terms, -1118; find the first term and the number of terms.

9. In a geometrical progression, given the first term, $\frac{2}{3}$; the ratio, $-\frac{1}{2}$; and the number of terms, 7; find the sum of the terms.

10. Develop by Binomial Formula $(a^2 b - \frac{1}{3} x a^{-2})^4$.

SEPTEMBER, 1885.

1. Find the least common multiple of $2x^3 - 3x^2 - x + 1$ and $6x^3 - x^2 + 3x - 2$.

2. $\frac{x}{2} - \frac{a - bcx}{2bc} = \frac{x}{6c} - \frac{ac - 4bx}{3bc}$. Find value of x.

3. A number is compounded of three figures whose sum is 17. The figure of the units is two thirds that of the hundreds. When 297 is subtracted the order of the figures is reversed. What is the number?

4. Multiply $3\sqrt{-1} - 2\sqrt{-2}$ and $4\sqrt{-2} - 2\sqrt{-1}$.

5. Reduce to an equivalent fraction with a rational denominator

$$\frac{\sqrt{a^2-1} - \sqrt{a^2+1}}{\sqrt{a^2-1} + \sqrt{a^2+1}}.$$

6. $\sqrt{3x} + \sqrt{3x+13} = \frac{91}{\sqrt{3x+13}}$. Find value of x.

7. $(2x-5)^2-(2x-1)^2=8x-5x^2-5$.

8. In an arithmetical progression, given the first term, $-\frac{2}{3}$; the number of terms, 18; and the last term, 5; find the common difference and sum of terms.

9. In a geometrical progression, given last term, -12; sum of terms, $-23\frac{1}{16}\frac{3}{2}$; and ratio, 2; find first term and number of terms.

10. Develop by Binomial Formula $(\frac{1}{2}ab^2-\frac{2}{3}a^2b^{-1})^5$.

JUNE, 1886.

1. Multiply $5x^{p-3}y^{r+3}-2x^{p-1}y^{r+1}-x^{p-2}y^{r+2}$ by $3x^{p+4}y^{r-1}+4x^{p+5}y^{r-2}-x^{p+3}y^r$.

2. Simplify $\left[\dfrac{1}{a+\dfrac{1}{b+\dfrac{1}{c}}}\div\dfrac{1}{a+\dfrac{1}{b}}\right]-\dfrac{1}{b[abc+a+c]}$.

3. Given $\begin{matrix}mx+2ny=p;\\ 2sx+ty=q.\end{matrix}$ Find the values of x and y.

4. The smaller of two numbers divided by the larger is .21 with a remainder of .04162. The greater divided by the smaller is 4 with .742 for a remainder. What are these numbers?

5. Given $\dfrac{x(2x-10)}{12}-\dfrac{(x-7)^2}{2}=\dfrac{(14-x)^2}{3}+(11-x)^2$, to find value of x.

6. $\dfrac{2}{x+\sqrt{2-x^2}}+\dfrac{2}{x-\sqrt{2-x^2}}=x$.

7. Expand $(2x^{\frac{2}{3}}+3x^{\frac{1}{3}}y)^4$.

8. Find sum of terms in a geometrical progression.

SEPTEMBER, 1886.

1. $x - \left\{\frac{2y-3}{4} + \frac{3x-5}{6}\right\} = 2 - \frac{2x-3y-1}{12}$;

$y + \left\{\frac{2x-y}{4} - \frac{3y-2}{3}\right\} = 1\frac{1}{6} - \frac{3-2x}{6}$.

Find values of x and y.

2. $\sqrt{a+x} + \sqrt{a-x} = \sqrt{6}$. Find values of x.

3. A number is compounded of three figures whose sum is 17. The figure of the hundreds is double that of the units. When 396 is subtracted the order of the figures is reversed. What is the number?

4. $3x^2 - 4x - 4 = 0$. Find values of x.

5. Find sum of six terms of the geometrical progression of which $\frac{8}{3}$ is the first term and $\frac{8}{9}$ the second term.

JUNE, 1887.

1. $5y - \frac{3x+7}{2} = 13\frac{1}{2}$,

$\frac{4x-3}{3} - \frac{2x+3y}{4} = -5\frac{3}{8}$. Find values of x and y.

2. $2x - 3y = 8$.
$y - 3z = -11$.
$x - 2y + 4z = 17$. Find values of x, y, and z.

3. A boy spent his money in oranges. If he had bought 5 more, each orange would have cost a half-cent less; if 3 less, a half-cent more. How much did he spend, and how many did he buy?

4. Multiply $\sqrt{p+q}+\sqrt{p-q}$ by $\sqrt{p+q}-\sqrt{p-q}$.

5. Multiply $\sqrt{-b}+a$ by $\sqrt{-b}-\sqrt{-a}$.

6. $7x-3x^2+14=0$. Complete the square and find the value of x.

7. $\sqrt{x+3}+\sqrt{3x-3}=10$. Find the value of x.

8. In an arithmetical progression there are given the first term, 4; the number of terms, 10; and the sum of the terms, 175. Find common difference and the last term.

9. Expand by the Binomial Formula $(2a^{\frac{2}{3}}-3b^{\frac{3}{4}})^5$.

SEPTEMBER, 1887.

1. Given $3x-\frac{4y-6}{3}=4-\frac{2x-4}{5}$,

$$2y-\frac{3x-2}{4}=5\tfrac{1}{2}-\frac{3y-5}{8},$$

to find values of x and y.

2. Add $\sqrt[3]{16x^4y^3}$, $\sqrt[6]{4x^8y^6}$, and $6xy\sqrt[9]{8x^3}$.

3. A man bought a certain number of eggs for 2 dollars. If he had paid 5 cents more per dozen, he would have received two dozens less for the same money. How many dozens did he buy, and what did he pay per dozen?

4. $2\sqrt{2x-3}-\sqrt{3x-7}=\sqrt{4x-11}$. Find values of x.

5. $3x^2-4x=55$. Find values of x.

6. In an arithmetical progression, given the first term, 3; the number of terms, 15; the sum of the terms, -165; to find the common difference and last term.

7. Expand $(2x-3y^2)^5$ by the Binomial Formula.

JUNE, 1888.

(Omit one from each set.)

I.

1. Resolve $64\,x^7 - x\,y^6$ into five factors.

2. Simplify $\dfrac{\dfrac{x^2}{y} + \dfrac{y^2}{x}}{x + \dfrac{y^2}{x} - y} \div \left(1 + \dfrac{x}{y}\right)$.

3. Given $\dfrac{a}{x} + \dfrac{b}{y} = c$, $\dfrac{a'}{x} + \dfrac{b'}{y} = c'$, to find values of x and y.

II.

1. Add $3\,x\sqrt{a^3 - a^2\,x}$, $-4\,a\sqrt{4\,a\,x^2 - 4\,x^3}$, and $5\sqrt{a^3\,x^2 - a^2\,x^3}$.

2. Multiply $2\sqrt[3]{a} - \sqrt{-x}$ by $3\sqrt{-a} + 2\sqrt[3]{x}$.

3. Given $\dfrac{x+2}{x-1} - \dfrac{4-x}{2\,x} = \dfrac{7}{3}$, to find values of x.

III.

1. Given $2\,x^2 - 3\,x\,y + y^2 = 35$, $2\,x - 3\,y = 13$, to find values of x and y.

2. Expand $(2\,a - 3\,b)^4$ by Binomial Formula.

3. In an arithmetical progression, given the first term, 14; the number of terms, 7; the sum of the terms, $59\frac{1}{2}$; find the common difference and last term.

SEPTEMBER, 1888.

1. Find value of

$$x\,(y + z) + y\,[x - (y + z)] - z\,[y - x\,(z - x)]$$

when $x = 3$, $y = 2$, $z = 1$.

2. A and B set out at the same time from the same spot to walk to a place 6 miles distant and back again. After walking for 2 hours, A meets B coming back. Supposing B to walk twice as fast as A, and each to maintain uniform speed throughout, find their respective rates of walking.

3. Solve the equation $\sqrt{x} + \sqrt{4+x} = \frac{4}{\sqrt{x}}$.

4. Solve the equation $\frac{5}{x+2} - \frac{2x-3}{2(x-2)} = -\frac{3}{6}$.

5. Find the sum of 10 terms of the geometrical progression in which the fourth term is 1 and the ninth term is $\frac{1}{243}$.

JUNE, 1889.

(Omit one from each set.)

I.

1. Find the lowest common multiple of $6x^3 + 11x^2 - 46x + 24$, and $12x^3 + 37x^2 - 42x + 8$.

2. Simplify

$$\frac{x-y}{(x+z)(y+z)} + \frac{y-z}{(x+y)(x+z)} - \frac{z-x}{(x+y)(y+z)}.$$

3. Solve $\dfrac{\frac{3x}{4} - \frac{y}{3}}{\frac{1}{2}} - \dfrac{\frac{x}{2} + \frac{2y}{5}}{\frac{13}{4}} = -\frac{7}{6}$.

$$2y - 3x = 23.$$

II.

1. A and B run a race of 480 feet. The first heat, A gives B a start of 48 feet, and beats him by 6 seconds; the second heat, A gives B a start of 144 feet, and is beaten by 2 seconds. How many feet can each run in a second?

2. Solve $\sqrt{3x+10} - \sqrt{3x+25} = -3$.

3. Solve $\dfrac{2x^2+3x-5}{3x^2+4x-1} = \dfrac{2x^2-x-1}{3x^2-2x+7}$.

III.

1. In an arithmetical progression, given the first term, -3; the common difference, $2\frac{1}{3}$; and the sum of the terms, 143; to find the last term and the number of terms.

2. In a geometrical progression, prove the formula for the sum of n terms.

3. Solve $2x^2 - 3y^2 = 60$, and $3x^2 - 4xy + y^2 = 64$.

September, 1889.

(Omit any one.)

1. Reduce to its simplest form,

$$3c^2 + c\,(2a - [5c - \{3a + \overline{c - 4a}\}]).$$

2. Find highest common factor of

$x^5 - x^4 - 5x^3 + 2x^2 + 6x$ and $x^5 + x^4 - x^3 - 2x^2 - 2x$.

3. Given $ax + by = c$, $px + qy = r$. Find values of x and y in terms of the other quantities.

4. Given $3x - \dfrac{2x+4}{3x-5} = 4 - \dfrac{2x-3}{2}$.

5. Divide $x^3 - y^2$ by $\sqrt{x} + \sqrt[3]{y}$.

EXAMINATIONS IN ALGEBRA FOR ADMISSION TO MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

JUNE, 1878.

1. Factor $6a^2x^3 - 15abx^2$ and $(2x-a)^2 - b^2$.

2. Solve the equation $(x-3)(2x+1) = 2(x+1)^2 - 14$.

3. Find the least common multiple of $12ab^2c$ and $15a^2d^4$.

4. Find the sum of $\frac{2a+2b}{a-b}$ and $\frac{2a-2b}{a+b}$.

5. Solve the equations $\left\{\begin{array}{rrrr} x-y- & z= & 10 \\ 2x+y- & z= & 11 \\ x+y+2z= & & -3 \end{array}\right\}$.

6. A number consisting of two figures is 4 times the sum of its digits; and if 27 be added to the number, the order of the digits will be inverted. What is the number?

7. Solve the equation $\frac{4}{x+1} + \frac{5}{x+2} - \frac{12}{x+3} = 0$.

8. Extract the square root of $x^{-8} - 6x^{-5} + 11x^{-2} - 6x + x^4$.

9. Solve the equation $\sqrt{x} + \sqrt{a+x} = \frac{2a}{\sqrt{a+x}}$.

SEPTEMBER, 1878.

1. Factor $8a^3x^2 - 18a^7$, also $x^2 - 11x + 30$.

2. Reduce $3a + \frac{11a-10}{15} - 2a - \frac{3a-5}{7}$ to a single fraction.

3. Divide $\frac{x^2-ax}{ax+a^2}$ by $\frac{ax^2-a^3}{x^2+ax}$, and reduce the result to its simplest form.

4. Solve the equation $\frac{x-3a}{2}+\frac{x}{3}=20a-\frac{x+19a}{2}$.

5. Solve the equations $\left.\begin{cases}\frac{x}{3}+3y=21\\ \frac{y}{3}-3x=-25\end{cases}\right\}$.

6. Simplify the following expressions:

$\sqrt{729\,a^6x^2}$, the cube root of $216\,a^{3n}c^6$, $\left(x^{\frac{3n}{2}}y^{\frac{5}{6}}\right)^6$.

7. Add $\sqrt{32}$ and $\sqrt{18}$. Multiply the cube root of $12a^2x^2$ by the cube root of $18ax^4$, and express the result in the simplest form.

8. Solve the equation $\frac{x+1}{x+2}-\frac{x-2}{x-1}=\frac{1}{6}$.

9. Divide 10 into two such parts that their product shall be 12 times their difference.

JUNE, 1879.

1. Divide $3x^3+11x-2x^4+12+4x^2$ by $x+4+2x^2$.

2. Factor $27x^3-125$.

3. Simplify the fraction $\dfrac{\frac{x+2y}{x+y}+\frac{x}{y}}{\frac{x}{x+y}-\frac{x+2y}{y}}$.

4. A said to B, "Nine years ago my age was to yours as 3 to 5." B replied, "In twelve years my age will be to yours as 4 to 3." Required their ages five years ago.

5. Simplify $\frac{(-8)^{\frac{1}{3}}}{9^{-\frac{5}{2}}}$.

6. Extract the square root of

$$x^3y^{-\frac{3}{2}}-4x^{\frac{3}{2}}y^{-\frac{1}{2}}+6-4x^{-\frac{3}{2}}y^{\frac{1}{2}}+x^{-3}y^3.$$

7. Subtract $\sqrt[3]{48}$ from $\sqrt[3]{162}$, multiply the remainder by $\sqrt[3]{4}$, and reduce the product to its simplest form.

8. Solve the equation $\dfrac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}} = \dfrac{x}{2}$.

9. Find two numbers whose difference multiplied by the greater produces 35, and whose sum multiplied by the less produces 18.

SEPTEMBER, 1879.

1. Reduce $\dfrac{x}{1+x} - \dfrac{x}{1-x} + \dfrac{x^2-1}{x^2}$ to its lowest terms.

2. Solve the equation

$$(a-mx)^2-(n-mx)^2-(a^2-an) = 0.$$

3. What principal, when put at simple interest for a years, at b per cent, will amount to c dollars?

4. Divide $2-4x^{-\frac{1}{2}}y^{\frac{1}{2}}+2x^{-\frac{1}{2}}y^{\frac{3}{2}}$ by $2+x^{\frac{1}{2}}y^{-\frac{1}{2}}+x^{-\frac{1}{2}}y^{\frac{1}{2}}$.

5. Extract the seventh root of $8\sqrt{2}$, reducing the result to its simplest form.

6. Reduce $\dfrac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}+\sqrt{1+x}}$ to an equivalent fraction with a rational denominator.

7. Solve the equation $\dfrac{2}{3-x} - \dfrac{3-x}{2} = \dfrac{5}{6}$.

8. Solve the equation $x^2-2\sqrt{x^2+4x-5} = 13-4x$.

9. The fore-wheel of a carriage makes 25 more revolutions than the hind-wheel in going 300 yards; but if the circumference of each wheel were increased by one yard, the fore-wheel would only make 15 revolutions more than the hind-wheel in going the same distance. Required the circumference of each wheel.

JUNE, 1880.

1. Find the value of

$$\frac{x^{\frac{3}{2}}y^{-2}}{a^{-\frac{3}{2}}b^{\frac{5}{4}}},\ \text{when } x=4,\ y=9,\ a=y^2,\ \text{and } b=x^2.$$

2. Substitute $y-3$ for x in $x^3+2x^2-15x-36$, and arrange the result.

3. Reduce $\frac{a}{a+b}+\frac{b}{a-b}+\frac{2ab}{a^2-b^2}$ to its simplest form.

4. Reduce $\frac{a^2+2+a^{-2}}{a^2-a^{-2}}$ to $\frac{a^2+1}{a^2-1}$.

5. Solve the equation $\frac{3x+2}{x-1}+\frac{2x-4}{x+2}=5$.

6. Add $\sqrt[3]{250}$ and $\sqrt[3]{16}$, multiply the sum by $\frac{1}{7}\sqrt{3}$, and find the sixth power of the product.

7. Solve the equation $\sqrt{x+13}-\sqrt{x-2}=3$.

8. Solve the equation $\frac{2}{3x}-\frac{x-1}{7-x}=0$.

SEPTEMBER, 1880.

1. Find the value of $a+b(x+y)^{\frac{1}{2}}-(a-b)(x-y)^{-\frac{1}{2}}$ when $a=5,\ b=1,\ x=12$, and $y=4$.

2. Multiply together $\frac{x^2-1}{x^2+3x}$, $\frac{x^2+6x+9}{x^2-x}$, and $\frac{x^2}{x+3}$, and reduce the result to its simplest form.

3. Solve the equation $\frac{x-2}{x}=\frac{x+4}{x+1}-\frac{9}{2x}$.

4. Solve the equations

$$\frac{x+y}{2}-\frac{x-y}{3}=8 \quad \text{and} \quad \frac{x+y}{3}+\frac{x-y}{4}=11.$$

5. Find the product of $\left(\frac{bx}{y}\right)^{\frac{1}{2}}$, $\left(\frac{ay}{x^2}\right)^{\frac{1}{3}}$, and $\left(\frac{x^2}{a^2b^2}\right)^{\frac{1}{4}}$.

6. Show that

$$(7+4\sqrt{3})(2-\sqrt{3}) = (2\sqrt{2}+\sqrt{6}-\sqrt{3}-2)(\sqrt{2}+1).$$

7. Solve the equation

$$\sqrt{x^4-3x^2+5} - \sqrt{x^4-5x^2+8} = 1.$$

8. Solve the equation $(2x-3)^2 = 8x$.

JUNE, 1881.

1. Remove the parentheses and reduce

$$2a - [5b + \{3c - (a + [2b - 3a + 4c])\}].$$

2. Factor the following expressions:

$$x^2 + 2xy + y^2 - 4 \text{ and } 9 - x^4 - 4y^2 + 4x^2y.$$

3. Find the greatest common divisor of

$$x^2 - 6x + 8 \text{ and } 4x^3 - 21x^2 + 15x + 20.$$

4. Find the least common multiple of

$$ax^2 + a^2x,\ x^2 - a^2,\ \text{and } x^3 - a^3.$$

5. Simplify $\dfrac{1}{x-1} - \overline{\dfrac{x}{x^2-1} + \dfrac{3}{x^3-1}}$.

6. Reduce $\dfrac{x - 3a + \dfrac{4a^2}{a+x}}{x - \dfrac{2a^2}{a+x}}$ to its simplest form.

7. Solve $\dfrac{3x-1}{4} - \dfrac{2x+1}{3} - \dfrac{4x-5}{5} = 4$.

8. Solve $\frac{3}{x}+\frac{1}{y}=\frac{5}{4}$, $\frac{2}{x}-\frac{3}{y}=1$, and state method of elimination.

9. Reduce $\frac{\sqrt{x}-4\sqrt{x-2}}{2\sqrt{x}+3\sqrt{x-2}}$ to an equivalent fraction having a rational denominator.

10. Solve the quadratic equations, $\sqrt{x}+\sqrt{x+5}=5$, $\frac{5}{x}-\frac{3x+1}{x^2}=\frac{1}{4}$, and $(x-5)^3-3(x-5)^{\frac{3}{2}}=40$.

SEPTEMBER, 1881.

1. Reduce $3x-\big(5x-[4x-(y-x)]\big)-(-x-3y)$ to its simplest form.

2. Factor the following expressions:

$$a^2-1, \qquad m^2-(x-y)^2, \qquad a^2-b^2+2bc-c^2.$$

3. Find the greatest common divisor of

$$x^4-x^3+2x^2+x+3 \quad \text{and} \quad x^4+2x^3-x-2.$$

4. Simplify the following expressions:

$$\frac{a}{a+b}+\frac{b}{a-b}+\frac{2ab}{a^2-b^2}, \text{ and } \frac{1}{x+y}+\frac{1}{x-y}-\frac{2x}{x^2+y^2}.$$

5. Divide $9+\frac{5y^2}{x^2-y^2}$ by $3+\frac{5y}{x-y}$ and reduce to the simplest form.

6. Solve the equation $\frac{2}{x-1}-\frac{3}{x+1}=\frac{1}{x^2-1}$.

7. Solve the simultaneous equations:

$$\frac{2}{x+y}+\frac{2}{x-y}=1, \qquad \frac{3}{x+y}-\frac{2}{x-y}=0.$$

8. Multiply $2x^{\frac{3}{2}}-3x^{\frac{1}{2}}-4+x^{-\frac{1}{2}}$ by $3x^{\frac{1}{2}}+x-2x^{\frac{3}{2}}$.

9. Solve the equation $\sqrt{x}-\sqrt{x-3}=\frac{2}{\sqrt{x}}$.

10. Solve the quadratic equations

$$\frac{x}{5-x} - \frac{5-x}{x} = \frac{15}{4}.$$

$$x^6 - 6x^3 = 16.$$

$$(3x+1)(4x^2-25) = 0.$$

JUNE, 1882.

1. Factor the following expressions:

$$4x^2 - 12xy + 9y^2, \qquad x^2 + 5x + 6, \qquad x^3 - 8y^3.$$

2. Find the greatest common divisor of

$$2x^3 - 4x^2 - 13x - 7 \quad \text{and} \quad 6x^3 - 11x^2 - 37x - 20.$$

3. Find the least common multiple of

$$4(1+x),\ 4(1-x),\ \text{and}\ 2(1-x^2).$$

4. Simplify $\dfrac{2a+b}{a-b} - \dfrac{2a-b}{a+b} - \dfrac{6ab}{a^2-b^2}$.

5. Multiply $a^n + a^{\frac{m}{2}}$ and $\sqrt{a}$ together.

6. Solve the equation $\dfrac{ax^2}{b-cx} + a + \dfrac{ax}{c} = 0.$

7. Solve the simultaneous equations

$$\frac{x-4}{5} - \frac{y+2}{10} = 0, \quad \text{and} \quad \frac{x}{6} + \frac{y-2}{4} = 3.$$

8. Extract the square root of

$$x^4 - 2x^3y + 3x^2y^2 - 2xy^3 + y^4.$$

9. Solve the quadratic equation

$$x - \frac{14x-9}{8x-3} = \frac{x^2-3}{x+1}.$$

10. Solve the simultaneous quadratic equations

$$\frac{1}{x} + \frac{1}{y} = 5, \quad \text{and} \quad \frac{1}{x^2} + \frac{1}{y^2} = 13.$$

SEPTEMBER, 1882.

1. Factor the following:

$$9m^2 - 24m + 16;\ x^2 - 2xy + y^2 - z^2.$$

2. Find the greatest common divisor of

$$12x^3 - 9x^2 + 5x + 2 \text{ and } 24x^2 + 10x + 1.$$

3. Find the least common multiple of

$$x^2 - 1;\ x^2 + 2x - 3;\ 6x^2 - x - 2.$$

4. Simplify $\dfrac{3}{x-a} + \dfrac{4a}{(x-a)^2} - \dfrac{5a^2}{(x-a)^3}$.

5. Show that

$$(a + b\sqrt{-1})(a - b\sqrt{-1}) = (a + b + \sqrt{2ab})(a + b - \sqrt{2ab}).$$

6. Solve the equation $\dfrac{x^2 - a}{bx} - \dfrac{a - x}{b} = \dfrac{2x}{b} - \dfrac{a}{x}$.

7. Solve the simultaneous equations

$$\frac{x+y}{y-x} = \frac{15}{8};\ 9x - \frac{3y+44}{7} = 100.$$

8. Extract the cube root of

$$x^6 + 3x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1.$$

9. Solve the quadratic equations

$$\frac{a}{3} + \frac{5x}{4} - \frac{x^2}{3a} = 0, \text{ and } 19x^4 + 216x^7 = x.$$

10. Solve the simultaneous quadratic equations

$$\frac{x}{a} + \frac{y}{b} = 1 \text{ and } \frac{a}{x} + \frac{b}{y} = 4.$$

JUNE, 1883.

1. Resolve into factors

$$1 - x^2 - y^2 + 2xy, \text{ and } 9x^2 - \frac{4y^4}{25}.$$

2. Find the greatest common divisor of

$$x^2 - 1, \; x^3 - 1, \text{ and } x^2 + x - 2.$$

3. Find the least common multiple of

$$x^2 - 1, \; x^2 + 2x - 3, \text{ and } x^2 + 4x + 3.$$

4. Reduce the following fractions to their simplest form:

$$x + 5 - \frac{2x - 15}{x - 3}; \qquad \frac{x}{1 + \frac{1}{x}} + 1 - \frac{1}{x + 1}.$$

5. A fraction which is equal to $\frac{2}{3}$ is increased to $\frac{8}{11}$ when a certain number is added to both its terms, and is diminished to $\frac{5}{7}$ when the same number plus one is subtracted from both. Find the fraction.

6. Solve the quadratic

$$\frac{x^2 - 4mnx}{(m + n)^2} = (m - n)^2.$$

7. Find the quadratic whose roots are -1 and $\frac{3}{4}$.

8. *Proportion.*

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h}$, prove that $\frac{a + c + e + g}{b + d + f + h} = \frac{a}{b}$.

9. *Arithmetical Progression.*

Find the sum of $a + 4a + 7a + \cdots$ to 10 terms.

10. *Geometrical Progression.*

Find the sum of $8 + 4 + 2 + \cdots$ to 10 terms.

11. *Binomial Theorem.*

Expand $(x - 3)^6$.

SEPTEMBER, 1883.

1. Find the value of

$$\left(\frac{x-a}{x-b}\right)^3 - \frac{x-2a+b}{x+a-2b} \text{ for } x = \frac{a+b}{2}.$$

2. Find the greatest common divisor of

$$3x^3 - 3x^2y + xy^2 - y^3 \text{ and } 4x^2y - 5xy^2 + y^3$$

3. Solve the equation

$$\frac{1}{x-a} - \frac{1}{x-b} = \frac{a-b}{x^2-ab}.$$

4. Solve the simultaneous equations

$$\frac{x-a}{b} + \frac{y-b}{a} = 0, \text{ and } \frac{x+y-b}{a} + \frac{x-y-a}{b} = 0.$$

5. Solve the quadrate $4a^2x = (a^2 - b^2 + x)^2$.

6. Solve the simultaneous equations

$$x^2 + 3xy = 54 \text{ and } xy + 4y^2 = 115.$$

7. If $\frac{a}{b} = \frac{c}{d}$, show that $\frac{a}{c} = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$.

8. *Arithmetical Progression.*

Find the sum of 15 terms of the series $\frac{1}{2}, -\frac{2}{3}, -1\frac{1}{6}, -\cdots$

9. *Geometrical Progression.*

Sum the series $6, -2, \frac{2}{3}, -\cdots$ to infinity.

10. *Binomial Theorem.*

Write the expansion of $(3 - 2x^2)^5$.

JUNE, 1884.

1. Divide $x^2y^2 - x^2 - y^2 + 1$ by $xy - x - y + 1$.

2. Find the G. C. D. and L. C. M. of the three following expressions:

$$3x^2 - 3x\,;\quad 4x^2 - 6x\,;\quad (2x - 2)\,(2x - 3).$$

3. Simplify

$$(a + b - 1)\left(\frac{1}{a} + \frac{1}{b} - 1\right) + \frac{(a + b)\,(a - 1)\,(b - 1)}{ab}.$$

4. Solve each of the two following equations:

$$(a)\quad \frac{3x - 5}{4} - \frac{x + 1}{7} = 2\,;\quad (b)\quad \sqrt{x} + \sqrt{a + x} = \frac{a}{\sqrt{x}}.$$

5. Solve the simultaneous equations

$$ax + by = 2\,;\quad ab\,(x + y) = a + b.$$

6. Simplify $\dfrac{(a^{p-q})^{p+q}\,(a^q)^{q+r}}{(a^p)^{p-q}}$.

7. Solve the equation

$$7 - \frac{x - 5}{x - 3} + \frac{4}{(x - 1)\,(x - 3)} = 0.$$

8. Solve the simultaneous equations,

$$(x + 1)\,(y + 2) = 10\,;\quad xy = 3.$$

9. Find two numbers such that their sum, their difference, and the sum of their squares may be to each other as 4, 1, and 17.

10. Show that if b is a mean proportional between a and c,

$$(a^2 + b^2)\,(b^2 + c^2) = (ab + bc)^2.$$

11. Expand by the Binomial Theorem $(a + 2\sqrt{a})^5$.

SEPTEMBER, 1884.

1. Simplify $\dfrac{\dfrac{2y+4x^2}{y^2}}{1+\dfrac{4x^2}{y^2}}$, after substituting $1-x^2$ for y.

2. Resolve $a^{12}-b^{12}$ into six factors.

Solve the following equations:

3. $\dfrac{x}{a}+\dfrac{y}{b}=1, \quad \dfrac{x}{b}-\dfrac{y}{a}=1.$

4. $\dfrac{x^2+2x}{2}-\dfrac{x^2-x+2}{4}+\dfrac{x-2}{3}=0.$

5. $\sqrt{2x+3}-\sqrt{x+1}=\sqrt{3x-8}.$

6. Prove that the square of half the sum of any two unequal numbers is less than half the sum of their squares.

7. Expand by the Binomial Theorem $\left(a-\dfrac{2b^2}{a}\right)^4$.

8. Insert two arithmetical means between 24 and 81. Also insert two geometrical means between the same numbers.

JUNE, 1885.

1. Divide a^3-a^2 by $a^{\frac{1}{2}}-a^{\frac{1}{3}}$.

2. Factor $x^2-x-30, \quad (x-y)^3-y^3, \quad x^{4n+1}-x.$

3. Find the value of $\dfrac{x^2-y^2}{x^2+y^2}$, when $x=\dfrac{a+b}{a-b}$ and $y=\dfrac{a-b}{a+b}$.

Solve the following equations:

4. $\dfrac{x-a}{x-b}+2\dfrac{x-b}{x-c}=3.$

5. $\dfrac{x+3x^{-1}}{1+2x^{-1}}=3-x.$

6. $x-\sqrt[3]{x^3-2x^2}=2.$

7. $(x+2)(y-3)=10, \quad xy=15.$

8. Find the cube of $1+\sqrt{-3}$.

9. The sum of three terms in arithmetical progression beginning with $\frac{3}{2}$ is equal to the sum of three terms in geometrical progression beginning with $\frac{3}{2}$, and the common difference is equal to the ratio. What are the two series?

SEPTEMBER, 1885.

1. Factor $x^2-6x-16$ and $1-9a+8a^2$.

2. Find highest common factor of x^2+x-6 and $2x^2-11x+14$.

3. Simplify $\dfrac{x}{x-y}+\dfrac{3x}{x+y}-\dfrac{2xy}{x^2-y^2}$.

Solve the following equations:

4. $\dfrac{x}{2}+\dfrac{1-2ax}{2a}+\dfrac{2x-1}{a^2}=0.$

5. $\dfrac{7}{x^2-4}-\dfrac{3}{x+2}=\dfrac{22}{5}.$

6. $\sqrt{x-32}+\sqrt{x}=16.$

7. Find the sum of 16 terms of the arithmetical progression $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \ldots$

8. Find the sum to infinity of the geometrical series $1, -\frac{a^2}{x^2}, \frac{a^4}{x^4}, \ldots$

9. Expand $(3x - 2y)^5$ by the Binomial Theorem.

JUNE, 1886.

1. Find the value of $\frac{x-a}{b} - \frac{x-b}{a}$, when $x = \frac{a^2}{a-b}$.

2. Add together $\frac{x-a}{(x+a)^2}$, $\frac{x+a}{(x-a)^2}$, $\frac{2x(3a-x)}{(x-a)(x+a)^2}$.

3. Solve $\frac{1}{2x-1} + \frac{2}{4x-3} = \frac{1}{x-1}$.

4. Solve $x + \sqrt{x^2 - a^2} = b$.

5. Show that $\frac{x^n - \sqrt{x^{2n} - 4}}{2}$ is the reciprocal of $\left(\frac{\sqrt{x^n+2} + \sqrt{x^n-2}}{2}\right)^2$.

6. Show that $(-1 + \sqrt{-3})^3 + (-1 - \sqrt{-3})^3 = 16$.

7. Solve $\frac{6}{x} + \frac{x}{6} = \frac{5(x-1)}{4}$.

8. Solve $x^2 + xy = 15$, $xy - y^2 = 2$.

9. Find the 4th term of $(a - 2b)^{10}$.

10. How many terms of $16 + 24 + 32 + 40 + \ldots$ amount to 1840?

SEPTEMBER, 1886.

1. Simplify $\dfrac{x+1}{x^2-2x}+\dfrac{2x-1}{x^2-x-2}-\dfrac{3x+2}{x^2+x}$.

2. Resolve $a^{12}-b^{12}$ into its prime factors.

3. Solve $\dfrac{x-5}{6}+\dfrac{5x-7}{9}-\dfrac{3x-7}{4}=\dfrac{5-x}{3}$.

4. Find the continued product of

$$\sqrt{a+b},\ \sqrt[4]{a-b},\ \text{and}\ \sqrt[4]{(a^2-b^2)^3}.$$

5. Extract the square root of $41+12\sqrt{5}$.

Solve the following equations:

6. $\dfrac{a}{x}+\dfrac{x}{b}+\dfrac{b}{a}=0.$

7. $\sqrt{x+\frac{1}{3}}=\sqrt[3]{x+\frac{1}{6}}.$

8. $\begin{cases}\sqrt{2x-y}=\sqrt{x-y}+1,\\ x^2+4y=17.\end{cases}$

9. There are two numbers whose geometrical mean is $\frac{4}{5}$ of their arithmetical mean; and if the two numbers be taken for the first two terms of an arithmetical progression, the sum of its first three terms is 36. What are the numbers?

JUNE, 1887.

1. Reduce to its lowest terms $\dfrac{x^4+3x-2}{x^4+3x^2+4}$.

2. Solve $\dfrac{a}{b-x}=\dfrac{b}{a-x}$.

3. Simplify $\left[\dfrac{(a^{m-n})^{m+n}\,(a^n)^{n+p}}{(a^m)^{m-n}}\right]^{\frac{1}{n}}$.

4. Solve $\sqrt{2x+1} - \sqrt{x+3} = \sqrt{x}$.

5. Solve $x^2 + y^2 = 20$, $x^2 - xy = 8$.

6. Solve $2x^3 + 8x^{-3} = 17$.

7. Reduce $\dfrac{5 + 3\sqrt{-1}}{1 + \sqrt{-1}}$ to the form $A + B\sqrt{-1}$.

8. The 1st term of an arithmetical progression is 2, and the difference between the 3d and 7th terms is 6. Find the sum of the first 12 terms.

SEPTEMBER, 1887.

1. Divide $x^2 + x^{-2}$ by $x^{\frac{2}{3}} + x^{-\frac{2}{3}}$.

2. Resolve into two factors $a^2 + b^2 - c^2 - d^2 + 2(ab + cd)$.

3. Solve $x - a + \sqrt{x^2 - 2ax} = b$.

4. Solve $\dfrac{3x - \sqrt{x^2 - 8}}{x - \sqrt{x^2 - 8}} = x + \sqrt{x^2 - 8}$.

5. Solve $(x - y)(x - 3y) = 24$, $x - 2y = 5$.

6. Form the quadratic equation whose roots are $a + b - c$ and $a - b + c$.

7. Insert three geometrical means between $3\frac{1}{3}$ and 18.

8. Give the first, third, and fifth terms of the expansion by the Binomial Theorem of $\left(x\sqrt{y} + \dfrac{y^2}{2\sqrt{x}}\right)^{10}$.

JUNE, 1888.

PRELIMINARY.

1. Factor $8cx - 12cy + 2ax - 3ay$,
and $2am - b^2 + m^2 + 2bn + a^2 - n^2$.

2. Find the G. C. D. & L. C. M. of $2x^4 - 11x^3 + 3x^2 + 10x$ and $3x^4 - 14x^3 - 6x^2 + 5x$.

3. Simplify $\dfrac{\dfrac{x+2y}{x+y} + \dfrac{x}{y}}{\dfrac{x+2y}{y} - \dfrac{x}{x+y}}$ and $\dfrac{1}{1 + \dfrac{1}{1 + \dfrac{1}{x}}}$.

4. Solve the equations

(*a*.) $\dfrac{5-2x}{x+1} = \dfrac{3-2x}{x+4}$,

(*b*.) $\dfrac{x}{2a} - 3 + \dfrac{x}{4a^3} = \dfrac{x}{3a^2} - 2a(2-3a)$.

5. At what time between 4 and 5 o'clock is the minute hand of a watch exactly 5 minutes in advance of the hour hand?

6. Solve the simultaneous equations

$$5x - 3y + 2z = 41.$$
$$2x + y - z = 17.$$
$$5x + 4y - 2z = 36.$$

7. Extract the square root of

$$x^2 + 4y^2 + 9z^2 - 4xy + 6xz - 12yz.$$

8. Reduce to an equivalent fraction having a rational denominator

$$\frac{\sqrt{x} - 4\sqrt{x-2}}{2\sqrt{x} + 3\sqrt{x-2}}.$$

FINAL.

1. Solve the equation $2x^2 + 3x - 5\sqrt{2x^2 + 3x + 9} = -3$.

2. Solve the simultaneous equations

$$\begin{cases} x^2 + xy + 4y^2 = 6. \\ 3x^2 + 8y^2 = 14. \end{cases}$$

3. Factor $x^4 - 7x^2y^2 + y^4$.

4. A person saves \$270 the first year, \$210 the second, and so on. In how many years will a person who saves every year \$180 have saved as much as he?

5. Expand $\left(m^{-\frac{3}{5}} + 2n^3\right)^7$.

Find 5th term of $\left(x^{-1} - 2y^{\frac{1}{2}}\right)^{11}$.

6. Form the equation whose roots are $-\frac{3}{5}$ and $\frac{4}{7}$.

7. Derive the formula for the sum of a series in geometrical progression.

8. Find three numbers in geometrical progression such that their sum shall be 14 and the sum of their squares 84.

COMPLETE.

1. Simplify $\dfrac{a^3 - b^3}{a^4 - b^4} - \dfrac{a - b}{a^2 - b^2} - \dfrac{1}{2}\left(\dfrac{a + b}{a^2 + b^2} - \dfrac{1}{a + b}\right)$.

2. Solve $\dfrac{x}{a + b} + \dfrac{y}{a - b} = 2, \quad x + y = 2a$.

3. Extract the square root of

$$4a^4 - 12a^3b + 29a^2b^2 - 30ab^3 + 25b^4$$

4. Simplify $\left(\dfrac{5 + 2\sqrt{3}}{4 - \sqrt{3}}\right)^2 \times \left(\dfrac{2 - \sqrt{3}}{\sqrt{3} + 1}\right)^2$.

5. Solve $\dfrac{9x-1}{x-\frac{1}{x}} = \dfrac{55}{6}$.

6. Solve $\sqrt{a+x} + \sqrt{a-x} = 2\sqrt{x}$.

7. Solve $x - \dfrac{x-y}{2} = 4$, $y - \dfrac{x+3y}{x+2} = 1$.

8. Find the sum of 18 terms of the series $\frac{2}{3}, -1, -2\frac{2}{3}, \ldots$

SEPTEMBER, 1888.

1. Reduce to its lowest terms $\dfrac{4x^2+3x-10}{4x^3+7x^2-3x-15}$.

2. Simplify $\left(\sqrt{\dfrac{a+x}{x}} - \sqrt{\dfrac{x}{a+x}}\right)^2 - \left(\sqrt{\dfrac{x}{a}} - \sqrt{\dfrac{a}{x}}\right)^2$.

3. A fraction becomes $\frac{3}{4}$ by the addition of 3 to the numerator and 1 to the denominator. If 1 be subtracted from the numerator and 3 from the denominator it becomes $\frac{1}{2}$. Find the fraction.

4. Solve $(x-a)^2 = (x-2a)(x^2+4a^2)^{\frac{1}{2}}$.

5. Form the quadratic equation whose roots are

$$\frac{(a+b)^2}{a-b} \text{ and } b-a.$$

6. Expand by the Binomial Theorem $\left(x+\dfrac{1}{x}\right)^7$.

7. Divide 111 into three parts such that the products of each pair may be in the ratios $4:5:6$.

8. Find the sum to infinity of the geometrical progression $\frac{3}{4}, \frac{1}{2}, \frac{1}{3}, \ldots$

MAY, 1889.

PRELIMINARY.

1. Find the greatest common divisor of $2x^4 - 12x^3 + 19x^2 - 6x + 9$ and $4x^3 - 18x^2 + 19x - 3$.

2. Simplify $\frac{a^3 - b^3}{a^3 + b^3} \times \frac{a + b}{a - b} \times \frac{(a^2 - ab + b^2)^2}{(a^2 + ab + b^2)^2}$.

3. One tap will empty a vessel in 80 minutes, a second in 200 minutes, and a third in 5 hours. How long will it take to empty the vessel if all the taps are opened?

4. Solve $\frac{x}{a + b} + \frac{y}{a - b} = 2a, \quad \frac{x - y}{4ab} = 1.$

5 Extract the square root of $x^4 - x^3 + \frac{x^2}{4} + 4x - 2 + \frac{4}{x^2}$.

6. Factor $2am - b^2 + m^2 + 2bn + a^2 - n^2$, and $2c^3m + 8c^2m - 42cm$.

7. Which is the greater, $\sqrt{10}$ or $\sqrt[5]{46}$, and why?

8. Extract the square root of $75 - 12\sqrt{21}$.

FINAL.

1. Write out the first four terms, the last four terms, and the middle term of $(x - 2y)^{14}$.

2. Find the sum of the first n terms of the series 1, 2, 3, ...

3. Find three geometrical means between 2 and 162.

4. Show that in the equation $x^2 + px + q = 0$, the sum of the roots is $-p$, and the product of the roots q.

5. Find the four roots of the equation $x^4 - 3x^2a^2 + a^4 = 0$.

6. A number consists of two figures whose product is 21; and if 22 is subtracted from the number and the sum of the squares of its figures added to the remainder, the order of the figures will be inverted. What is the number?

7. Solve $3x^2 + 15x - 2\sqrt{x^2 + 5x + 1} = 2$.

8. Form the equation whose roots are $(a - \frac{1}{2})$, $(b + \frac{2}{3})$.

COMPLETE.

1. Simplify

$$\left(x - \frac{xy - y^2}{x + y}\right)\left(x - \frac{xy^2 - y^3}{x^2 + y^2}\right) \div \left(1 - \frac{xy - y^2}{x^2}\right).$$

2. Reduce to its lowest terms $\dfrac{x^4 - 2x^3 + 2x - 1}{x^6 - 15x^2 + 24x - 10}$.

3. Simplify $\dfrac{(a+b)^{\frac{1}{2}} + (a-b)^{\frac{1}{2}}}{(a+b)^{\frac{1}{2}} - (a-b)^{\frac{1}{2}}} + \dfrac{(a+b)^{\frac{1}{2}} - (a-b)^{\frac{1}{2}}}{(a+b)^{\frac{1}{2}} + (a-b)^{\frac{1}{2}}}$.

4. A certain number when divided by a second gives a quotient 3 and a remainder 2; if 9 times the second number be divided by the first, the quotient is 2 and the remainder 11. Find the two numbers.

5. Solve $x^{\frac{1}{2}} - a^{\frac{1}{2}} = (x - b)^{\frac{1}{2}}$.

6. Solve $x^4 + 4abx^2 = (a^2 - b^2)^2$.

7. If A is the sum of the odd terms, and B of the even terms, in the expansion of $(x + a)^4$, show that $A^2 - B^2 = (x^2 - a^2)^4$.

8. If $x - y$ is a mean proportional between y and $y + z - 2x$, show that x is a mean proportional between y and z.

9. The second term of a geometrical progression is 54, and the fifth term 16. Find the series.

SEPTEMBER, 1889.

FINAL.

1. Solve (*a*.) $3x^2 - 53x + 34 = 0.$

(*b*.) $\sqrt{a+x} + \sqrt{a-x} = \sqrt{b}.$

2. For what value of m will the equation $2x^2 + 8x + m = 0$ have equal roots? For what value, imaginary roots?

3. Solve $x^2 + 3xy = 54, \quad xy + 4y^2 = 115.$

4. Find two numbers such that their sum, their difference, and the sum of their squares may be to each other as 4, 1, 17.

5. In a geometrical progression, given the number of terms 8, the ratio $\frac{1}{2}$, and the sum of the terms $7\frac{3}{16}$; find the last term and the first term.

6. Expand $\left(a^{\frac{2}{3}}y - b^{-\frac{1}{2}}\right)^8.$

7. Four numbers are in arithmetical progression. The product of the 1st and 3d is 27, and of the 2d and 4th, 72. What are the numbers?

COMPLETE.

1. Resolve into four factors

$$(x^2 + y^2 - z^2 - u^2)^2 - 4\,(xy - zu)^2.$$

2. Solve $\dfrac{x+2a}{x-2b} = \dfrac{(x+a)^2}{(x-b)^2}.$

3. Given $8x = \left(\dfrac{a}{b} + \dfrac{b}{a}\right)^3, \quad 8y = \left(\dfrac{a}{b} - \dfrac{b}{a}\right)^3;$

show that $x^{\frac{2}{3}} - y^{\frac{2}{3}} = 1.$

4. Solve $\dfrac{5}{x+3} + \dfrac{6}{x+4} = 2.$

5. Solve $x(y+3) = 5, \quad y(x+1) = 4.$

6. Expand by the Binomial Theorem $\left(a^{\frac{2}{5}} - 2b^{\frac{1}{5}}\right)^5.$

7. Solve $x^{\frac{1}{3}} + x^{-\frac{1}{3}} = \sqrt{5 - x^{-\frac{2}{3}}}.$

8. Find a series in arithmetical progression whose fourth term is 4, and the sum of the first ten terms, 50.